Reg. No………………………………………….   Name……………………………………………   **18U528**

B.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER/NOVEMBER 2018

SEMESTER **–5: MATHEMATICS (CORE COURSE)**

COURSE: **15U5CRMAT7: ABSTRACT ALGEBRA**

(Common for Regular 2016 admission & Supplementary 2015 admission)

Time: Three Hours   Max. Marks: 75

PART A

*Answer **All** Questions. Each Question carries **1** Mark*

1. State true or false: Null set forms a group.
2. Define symmetric group on n letters.
3. Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{pmatrix}$ as a product of transpositions.
4. Define isomorphism.
5. Which are the distinct left cosets of $2\mathbb{Z}$ in $\mathbb{Z}$?
6. State Lagrange's Theorem?
7. Define canonical map from $\mathbb{Z}$ to $\mathbb{Z}_n$.
8. Which are the zero divisors in $\mathbb{Z}_{12}$.
9. Define characteristic of a ring.
10. State true or false: Every subring is an ideal.   (1 x 10 = 10)

PART B

*Answer **Any Eight.** Each Question carries **2** Marks*

11. Prove the uniqueness of identity element in a group.
12. Define Klein - 4 group and draw its group table.
13. Show that if $a \in G$, where G is a finite group with identity e, then there exist a positive integer n such that $a^n = e$.
14. Prove: Every cyclic group is abelian.
15. Prove: Every group is isomorphic to itself.
16. Prove: Every group of prime order is cyclic.
17. Differentiate between unity and unit in a ring, with examples.
18. Prove: $\mathbb{Z}_n$ is a field, if n is prime.
19. Prove or disprove: An ideal is always a subring.
20. Prove $n\mathbb{Z}$ is an ideal of $\mathbb{Z}$ and hence find the factor ring $\mathbb{Z}/n\mathbb{Z}$.   (2 x 8 = 16)

PART C

*Answer **Any Five.** Each Question carries **5** Marks*

21. Prove that a cyclic group with only one generator can have atmost 2 elements.
22. Let S be the set of all real numbers except -1. Define * on S by $a * b = a + b + ab$. Show that (S, *) forms a group.
23. Find all left cosets of $\langle(1, 2)\rangle$ in $\mathbb{Z}_2 \times \mathbb{Z}_4$.

24. Define inner automorphism of G under conjugation by g, and prove that it is an automorphism of G.
25. State and prove characterisation of maximal normal subgroups.
26. Prove: In the ring $\mathbb{Z}_n$, the divisors of 0 are precisely those elements which are not relatively prime to n.
27. Prove: If a ring R can be partitioned into cells with both the induced operations well defined and if the cells form a ring under these induced operations, then the cell containing additive identity 0 of R will be a subgroup N of the additive group $(R,+)$. Furthermore, $\forall\, r \in R$ and $\forall\, n \in N$, both $rn \in N$ and $nr \in N$. (5 x 5 = 25)

PART D

*Answer **Any Two.** Each Question carries **12** Marks*

28. Prove: No permutation of a finite set can be expressed both as a product of an even number of transpositions and as a product of an odd number of transpositions.
29. State and prove Cayley's Theorem.
30. (a) Prove: Every field is an integral domain.

    (b) Prove: Every finite integral domain is a field.
31. Prove: " If $(N,+)$ is an additive subgroup of the additive group $(R,+)$ in a ring $(R,+,.)$, then the operations of induced addition and induced multiplication on $\{r+N : r \in R\}$ are both well defined iff $rN \subseteq N$ and $Nr \subseteq N, \forall\, r \in R$."

(12 x 2 = 24)

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