Reg. No…………………………………………….   Name……………………………………………… **18U504**

B. Sc. DEGREE END SEMESTER EXAMINATION OCTOBER/NOVEMBER 2018

SEMESTER **–5: MATHEMATICS**

(CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATION)

COURSE: **15U5CRMAT5-15U5CRCMT5: MATHEMATICAL ANALYSIS**

(Common for Regular 2016 admission & Supplementary 2015 admission)

Time: Three Hours   Max. Marks: 75

PART A

*Answer **all** questions. Each question carries **1** mark.*

1. Find the infimum and the supremum of the set {-1,$\frac{-1}{2}$,$\frac{-1}{3}$,$\frac{-1}{4}$,$\frac{-1}{5}$, … … …}
2. Define open interval.
3. Define neighborhood of a point a.
4. "The set **R** of real numbers is an open set." State whether this statement is true or false.
5. Define limit point of a set.
6. Define the convergence a sequence {S_n}.
7. Consider the sequence {S_n} where S_n=1 $\forall n \in N$. Give a limit point of this sequence, if it has one.
8. State Cesaro's theorem.
9. Write $\frac{1+2i}{3-4i}$ in the form a+bi
10. Find the centre and radius of $|z-1+3i|$ =2   (1 x 10 = 10)

PART B

*Answer **any eight** questions. Each question carries **2** marks.*

11. State Archimedean property of real numbers.
12. Prove that the greatest member of a set if it exists, is the supremum of the set.
13. Show that if a set S is bounded then so is its closure $\tilde{S}$.
14. Show that the set $S = \{x \,/\, 0 < x < 1, x \in R\}$ is open but not closed.
15. Define the derived set of a set S. Obtain the derived set of the open interval (a,b).
16. Find lim $\frac{2n-3}{n+1}$
17. Show that for any real number x, lim $\frac{x^n}{n!}$ = 0
18. Prove that every monotonic increasing sequence which is not bounded above diverges to +∞
19. If z is point such that $|z| < 2$, show that $|z^3 + 3z^2 - 2z + 1| < 25$
20. What is the principal argument Arg (-1-i)? Find also arg (-1-i).   (2 x 8 = 16)

PART C

*Answer **any five** questions. Each question carries **5** marks.*

21. Prove that every open interval (a, b) contains a rational number.
22. Prove that a non-empty finite set is not a neighborhood of any point.

23. Show that the set of rational numbers in the closed interval [0,1] is countable.
24. Prove that every convergent sequence is bounded.
25. Prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$
26. Show that the sequence $\{b_n\}$ where
    b_n=$\left\{\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \frac{1}{(n+3)^2} + \cdots + \frac{1}{(2n)^2}\right\}$, converges to zero.
27. Find all the values of $(-8i)^{\frac{1}{3}}$ (5 x 5 = 25)

PART D

*Answer **any two** questions. Each question carries **12** marks.*

28. Prove that the set of rational numbers is not order –complete.
29. State and prove Bolzano-Weierstrass Theorem for sets.
30. State and prove Cauchy's general principle of convergence.
31. (i) If $\{a_n\}$ be a sequence such that $lim \frac{a_{n+1}}{a_n} = l$ where $|l| < 1$, then show that $lim a_n = 0$.

    (ii) Show that the sequence $\{S_n\}$ where S_n=$\left(1 + \frac{1}{n}\right)^n$ is convergent. And that the limit lies between 2 and 3. (12 x 2 = 24)

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