Q. Code: **P 225** Reg. No………………………… Name: ……………………………………………

M SC DEGREE END SEMESTER EXAMINATION – APRIL/MAY 2015

SEMESTER -2: M SC MATHEMATICS

COURSE: P2MATT08- ADVANCED COMPLEX ANALYSIS

Time: 3Hours Max Marks: 75

PART A

Answer any FIVE questions; 2 marks each

1. Show that $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$.
2. Prove that every function which is meromorphic in the whole plane is the quotient of two entire functions.
3. State Hadamard's theorem.
4. Write a note on the zeros of the Zeta function.
5. State Harnack's Principle.
6. Define subharmonic function and give one example
7. Define Germs and Sheaves.
8. Prove that a non constant elliptic function has equally many poles as it has zeros.

PART B

Answer any FIVE questions; 5 marks each

9. Prove that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{-\infty}^{\infty} \frac{1}{(z-n)^2}$.

10. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

11. Show that for $\sigma = Re\ s > 1$, $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty} (1 - p_n{}^{-s})$, where p_n are the ascending sequence of primes and $\zeta(s)$ is Riemann's zeta function.

12. Prove that a family F is normal if and only if its closure $\bar{F}$ with respect to the distance function is compact.

13. Prove Harnack's inequality.

14. Prove that a continuous function $v(z)$ is subharmonic in Ω if and only if it satisfies the inequality $v(z_0) \le \frac{1}{2\pi}\int_0^{2\pi} v(z_0 + re^{i\theta})\, d\theta$ for every disk $|z - z_0| \le r$ contained in Ω.

15. Prove that the zeros $a_1, \ldots\ldots, a_n$ and poles $b_1, \ldots., b_n$ of an elliptic function satisfy $a_1 + \cdots + a_n \equiv b_1 + \cdots + b_n (mod\ M)$.

16. Show that any two bases of the same module are connected by a unimodular transformation.

PART C
10 marks each

17 (A) State and prove Mittag-Leffler's theorem

OR

(B) (i) Prove that $\frac{\pi}{sin\pi z} = \lim_{m\to\infty} \sum_{-m}^{m} (-1)^n \frac{1}{z-n}$.

(ii) Show that $\Gamma\left(\frac{1}{6}\right) = 2^{-\frac{1}{3}} \left(\frac{3}{\pi}\right)^{\frac{1}{2}} \Gamma\left(\frac{1}{3}\right)^2$.

18 (A) State and prove Arzela-Ascoli 's theorem.

OR

(B) Derive the functional equation for Riemann zeta function.

19 (A) State and Prove Riemann mapping theorem.

OR

(B) (i) Write notes on properties of subharmonic functions.

(ii) Prove Harnack's principle.

20. (A) (i) If u, v and w are complex numbers different from the poles of $\wp(z)$ such that $u + v + w = 0$. Then $\begin{vmatrix} \wp(u) & \wp'(u) & 1 \\ \wp(v) & \wp'(v) & 1 \\ \wp(w) & \wp'(w) & 1 \end{vmatrix} = 0.$

(ii) Prove that homotopy is an equivalence relation on the set of all maps from X to Y.

OR

(B) (i) Prove that $\wp(2z) = \frac{1}{4}\left(\frac{\wp''(z)}{\wp'(z)}\right)^2 - 2\,\wp(z).$

(ii) State and prove Legendre's relation.