Q. Code: **P237** Reg. No.............................. Name: ....

M SC DEGREE END SEMESTER EXAMINATION – APRIL/MAY 2015

SEMESTER -2: M SC MATHEMATICS

COURSE: P2MATT09- PARTIAL DIFFERENTIAL EQUATIONS

Time: 3Hours Max Marks: 75

PART A

Answer any FIVE questions; 2 marks each

1. Verify that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable.
2. Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.
3. Along every characteristic strip of the equation $F(x, y, z, p, q) = 0$, the function $F(x, y, z, p, q)$ is a constant.
4. Find a complete integral of the equation $zpq = p + q$.
5. Classify the second order partial differential equations and give example for each.
6. Reduce the equation $u_{xx} + x^2 u_{yy} = 0$ to a canonical form.
7. Show that the surfaces $x^2 + y^2 + z^2 = c,\ c > 0$ can form a family of equipotential surfaces.
8. Solve the one dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ by separating the variables.

PART B

Answer any FIVE questions; 5 marks each

9. Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1,\ z = 1$.
10. Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0,\ z = 1$.
11. Explain Charpit's method to find a complete integral of a first order partial differential equation , $f(x, y, z, p, q) = 0$.
12. Find a complete integral of the equation $p^2 x + q^2 y = z$ by Jacobi's method.

13. Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$=to canonical form and hence solve it.

14. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$

15. Write a note on Monge's method.

16. Prove that for the equation $\frac{\partial^2 u}{\partial x \partial y}+\frac{1}{4}u=0$, the Riemann function is $v(x,y;\alpha,\beta) = J_0(\sqrt{(x-\alpha)(y-\beta)})$ where J_0 denote the Bessel's function of the first kind of order zero.

PART C

10 marks each

17. (A) Prove that the Pfaffian differential equation $\vec{X}.d\vec{r}=0$ is integrable if and only if $\vec{X}.\text{curl}\,\vec{X}=0$.

OR

(B) Prove that if $u_i(x_1, x_2,\ldots,x_n, z) = c_i \quad (i = 1,2,3,\ldots\ldots, n)$ are independent solutions of the equations $\frac{dx_1}{P_1} = \frac{dx_2}{P_2} = \cdots . = \frac{dx_n}{P_n} = \frac{dz}{R}$, then the relation $\emptyset(u_1, u_2, \ldots\ldots, u_n) = 0$ in which the function $\emptyset$ is arbitrary, is a general solution of the linear partial differential equation

$$P_1\frac{\partial z}{\partial x_1}+ P_2\frac{\partial z}{\partial x_2} + P_3\frac{\partial z}{\partial x_3} + \cdots \ldots\ldots + P_n\frac{\partial z}{\partial x_n} = R.$$

18. (A) Using the method of characteristics find the solution of the equation $z = \frac{1}{2}(p^2+q^2) + (p-x)(q-y)$ which passes through the x axis.

OR

(B) Prove that the first order partial differential equations $f(x, y, z, p, q) = 0$ & $g(x, y, z, p, q) = 0$ are compatible if and only if $[f, g] \equiv 0$.

19. (A) Reduce the equation $y^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + x^2 \frac{\partial^2 u}{\partial y^2} = \frac{y^2}{x} \frac{\partial u}{\partial x} + \frac{x^2}{y} \frac{\partial u}{\partial y}$ to a canonical form and hence solve it.

OR

(B) If $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\emptyset_r(\xi)$ is an arbitrary function of the single variable ξ, then if $\alpha_r \neq 0$, U_r = exp$(\frac{-\gamma_r}{\alpha_r} x)\emptyset_r(\beta_r x - \alpha_r y)$ is a solution of the equation $F(D, D')z = 0$.

20.(A) Solve the equation $r + 4s + t + rt - s^2 = 2$ by Monge's method .

OR

(B) Define the equipotential surfaces. Show that the surfaces $x^2 + y^2 + z^2 = cx^{\frac{2}{3}}$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.