Q. Code: **P 237** Reg. No............................... Name: ....

M SC DEGREE END SEMESTER EXAMINATION – APRIL/MAY 2015

SEMESTER -2: M SC MATHEMATICS

COURSE: P2MATT09- PARTIAL DIFFERENTIAL EQUATIONS

Time: 3Hours Max Marks: 75

PART A

Answer any FIVE questions; 2 marks each

1. Prove that there always exists an integrating factor for a Pfaffian differential equation in two variables.
2. Find the integral surface of the equation $(2xy-1)p+(z-2x^2)q=2\,(x-yz)$ which passes through the line $x_0(s)=1, y_0(s)=0 \; and \; z_0(s)=s$.
3. Show that the equations $xp-yq=x$, $x^2p+q=xz$ are compatible and find their solution.
4. Find a complete integral of the equation $(p^2+q^2)y=qz$.
5. Solve the equation $\frac{\partial^4 z}{\partial x^4}+\frac{\partial^4 z}{\partial y^4}=2\,\frac{\partial^4 z}{\partial x^2 \partial y^2}$.
6. Define hyperbolic, parabolic and elliptic equations. Give example of a parabolic equation.
7. Define Riemann- Green function.
8. Write the Monge's equations for the non linear equation $y^2r-2ys+t=p+6y$.

PART B

Answer any FIVE questions; 5 marks each

9. Find the orthogonal trajectories on the cone $x^2+y^2=z^2\;tan^2\alpha$ of its intersections, with the family of planes parallel to $z=0$.
10. Verify that the differential equation $(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$ is integrable and find its primitive.
11. Find a complete integral of the partial differential equation $(p^2+q^2)x=pz$ and deduce the solution which passes through the curve $x=0,\; z^2=4y$.
12. Show that a complete integral of $f(u_x,u_y,u_z)=0$ is $u=ax+by+cz+d$ where $f(a,b,c)=0$. Hence find the complete integral of $u_x+u_y+u_z-u_xu_yu_z=0$.

13. If $u_1, u_2, \ldots \ldots, u_n$ are solutions of the homogeneous linear partial differential equation $F(D,D')z = 0$, then $\sum_{r=1}^{n} c_r\, u_r$ where c_r's are arbitrary constants, is also a solution.

14. Reduce the equation $\frac{\partial^2 u}{\partial x^2} = x^2 \frac{\partial^2 u}{\partial y^2}$ to canonical form.

15. The solution of a certain Neumann problem can differ from one another by a constant only.

16. Derive the condition that the surface $f(x, y, z) = c$ form a family of equipotential surfaces.

PART C

10marks each

17. (A) (i) Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$

orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

(ii) The general solution of the linear partial differential equation $Pp + Qq = R$ is

$F(u, v) = 0$ where F is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$

form a solution of the equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

OR

(B) Prove that the Pfaffian differential equation $\vec{X}.d\vec{r}=0$ is integrable if and only if

$\vec{X}.\text{curl}\,\vec{X}=0$.

18. (A) Find the complete integral of the equation $p^2x + qy = z$ and derive the equation of the integral surface containing the line $y = 1, x + z = 0$ is a generator.

OR

(B) Describe Jacobi's method . Solve the equation $z^2 + zu_z - u^2{}_x - u^2{}_y = 0$ by Jacobi's method.

19. (A) Solve the equation $r - 4s + 4t = e^{2x+y}$.

OR

(B) Reduce the equation $(n-1)^2 \frac{\partial^2 u}{\partial x^2} - y^{2n} \frac{\partial^2 u}{\partial y^2} = ny^{2n-1} \frac{\partial u}{\partial y}$ to canonical form and find its general solution.

20. (A) Describe Monge's method . Solve $r = t$.

OR

(B) Define Riemann function. Prove that for the equation $\frac{\partial^2 u}{\partial x \partial y} + \frac{1}{4} u = 0$, the Riemann function $(x, y\,; \alpha, \beta) = J_0 \sqrt{(x-\alpha)(y-\beta)}$ where $J_0(z)$ denote Bessel's function of the first kind of order zero.