Q. Code: **P225** Reg. No…………………………… Name: ……………………………………..……….

## M SC DEGREE END SEMESTER EXAMINATION – APRIL/MAY 2015

## SEMESTER -2: M SC MATHEMATICS

### COURSE: P2MATT08- ADVANCED COMPLEX ANALYSIS

Time: 3Hours Max Marks: 75

## PART A

Answer any FIVE questions; 2 marks each

1. Prove that the necessary and sufficient condition for the absolute convergence of the product $\prod_1^{\infty}(1+a_n)$ is the convergence of the series $\sum_1^{\infty}|a_n|$.
2. Prove that $\Gamma(z)\Gamma(1-z)$ is a meromorphic function without zeros.
3. State Arzela's theorem.
4. Prove that an entire function of fractional order assumes every finite value infinitely many times.
5. State Harnack's Principle.
6. If $v_1$ and $v_2$ are subharmonic in Ω, then $v = \max(v_1, v_2)$ is subharmonic in Ω.
7. Define Homotopy.
8. Prove that an elliptic function without poles is a constant.

## PART B

Answer any FIVE questions; 5 marks each

9. State and prove the Weierstrass theorem for infinite products.
10. Prove that $\sin \pi z = \pi z \prod_1^{\infty} \left(1-\frac{z^2}{n^2}\right)$.
11. Show that the function $\xi(s) = \frac{1}{2}\, s\,(1-s)\,\pi^{\frac{-s}{2}}\,\Gamma(\frac{s}{2})\,\zeta(s)$ is entire and satisfies $\xi(s) = \xi(1-s)$ where ζ(s) is Riemann Zeta function.
12. State and prove Jensen's formula.
13. Define subharmonic function with example and describe the properties of subharmonic functions.
14. Derive Schwarz- Christoffel formula.
15. Show that the Weierstrass $\wp$ function is the inverse of an elliptic integral.
16. State and prove Legendre's relation.

## PART C

10 marks each

17.(A) (i) Prove that the infinite product $\prod_1^\infty(1+a_n)$ with $1+a_n \neq 0$ converges simultaneously with the series $\prod_1^\infty \log(1+a_n)$ whose terms represent the values of the principal branch of the logarithm

(ii) Prove that $\pi cot\pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2z}{z^2-n^2}$.

**OR**

(B) Derive Legendre's duplication formula.

18. (A) Derive the functional equation for Riemann zeta function.

**OR**

(B) (i) Derive Poisson-Jensen formula.

(ii) Prove that the family $F$ is totally bounded if and only if to every compact set $E \subset \Omega$ and every $\varepsilon > 0$ it is possible to find $f_1, f_2, \ldots, f_n\ \varepsilon F$ such that every $f \varepsilon F$ satisfies $d(f, f_j) < \varepsilon$ on $E$ for some $f_j$.

19. (A) (i) Prove that a continuous function $u(z)$ which satisfies the condition

$u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + re^{i\theta})\, d\theta$ is necessarily harmonic.

(ii) Prove that a continuous function $v(z)$ is subharmonic in Ω if and only if it satisfies the inequality $v(z_0) \leq \frac{1}{2\pi}\int_0^{2\pi} v(z_0 + r\, e^{i\theta})\, d\theta$ for every disk $|z - z_0| \leq r$ contained in Ω.

**OR**

(B) State Harnack's principle by proving the corresponding Harnack's inequality.

20. (A) (i) Prove the addition theorem for the $\wp$ function:

$$\wp(u+v) = \frac{1}{4}\left\{\frac{\wp'(u) - \wp'(v)}{\wp(u) - \wp(v)}\right\}^2 - \wp(u) - \wp(v)$$

(ii) Show that any two bases of the same module are connected by a unimodular transformation.

**OR**

(B) (i) Prove that the zeros $a_1, \ldots\ldots, a_n$ and poles $b_1, \ldots., b_n$ of an elliptic function satisfy

$a_1 + \cdots + a_n \equiv b_1 + \cdots + b_n \ (mod\ M).$

(ii) Prove that a discrete module consists either of zero alone, of the integral multiples $n\omega$ of a single complex number $\omega \neq 0$, or of all linear combinations $n_1\omega_1 + n_2\omega_2$ with integral coefficients of two numbers $\omega_1, \omega_2$ with nonreal ratio $\frac{\omega_2}{\omega_1}$.