Q. Code: .... Reg. No. .... Name:........................

MSc DEGREE END SEMESTER EXAMINATION OCTOBER 2015
SEMESTER-1: MATHEMATICS
COURSE CODE - P1MATTO5: COMPLEX ANALYSIS-I

Time: 3 Hours Max. Marks: 75

Part A

Answer any 5 Questions. Each carries 2 marks

1. Show that a linear transformation has atmost two fixed points.
2. Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$? Also find the inverse of this transformation?
3. Compute $\int_{|z|=1} |z-1||dz|$ for the positive sense of the circle?
4. Show that if γ is a closed curve lies inside a circle C, then $n(\gamma, a) = 0$ for all points a outside the circle C?
5. Show that the functions e^z and $\sin z$ have essential singularities at ∞?
6. Show that if Ω is a simply connected region, then $n(\gamma, a) = 0$ for all cycles γ in Ω and all points $a \notin \Omega$?
7. How many roots of the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ are in $\{z : |z| < 1\}$?
8. State The Residue Theorem?

$2 \times 5 = 10$

Part B

Answer any 5 Questions. Each carries 5 marks

9. An analytic function in a region Ω *degenerates* if it reduces to a constant. Show that if f is analytic on a region Ω and $\arg f(z)$ is constant, then f degenerates in Ω.
10. If $Sz = \frac{z+2}{z+3}, Tz = \frac{z}{z+1}$, find STz, TSz and $S^{-1}Tz$?
11. Let $a \in \mathbb{C}$ and γ be a piecewise differentiable closed curve which doesn't pass through the point a. Show that the integral $\frac{1}{2\pi i}\int_{\gamma} \frac{dz}{z-a}$ takes integer values?
12. Let $f : \mathbb{C} \to \mathbb{C}$ be analytic, such that $f(\mathbb{C}) = \{f(z) | z \in \mathbb{C}\}$ is contained in the open unit disk. Show that f is a constant?
13. Show that a non-constant analytic function map open sets onto open sets?
14. Show that if $f(z)$ is analytic, non-constant and non-vanishing (that is, $f(z) \neq 0$ for all z) in a region Ω, the $|f(z)|$ has no minimum value in Ω?
15. Show that if a function u is harmonic in a disk $B(0;R)$ then $\frac{1}{2\pi}\int_{|z|=r} u\,d\theta$ is a constant for every $0 < r < R$?

16. Find the poles and residues of (a) $\frac{1}{\sin z}$ (b) $\frac{1}{z^m(1-z)^n}$, $m,n \in \mathbb{Z}^+$.

$5 \times 5 = 25$

Part C

Answer either (a) or (b) of the following four questions. Each carries 10 marks

17. (a) Show that the cross ratio (z_1, z_2, z_3, z_4) is real if, and only if the four points lie on a circle or on a straight line?

    (b) Describe the Riemann surface associated with the function

$$w = \frac{1}{2}\left(z + \frac{1}{z}\right).$$

18. (a) Suppose that $\phi(t)$ is continuous on the arc γ. For $n \in \mathbb{Z}^+$, define

$$F_n(z) = \int_\gamma \frac{\phi(t)}{(t-z)^n} dt.$$

Show that F_n is analytic in each of the regions determined by γ and its derivative $F_n'(z) = nF_{n+1}(z)$ for all z?

    (b) i. Prove that a function which is analytic in the whole plane and satisfies an inequality $|f(z)| < |z|^n$ for some n and all sufficiently large $|z|$ reduces to a polynomial?

    ii. State and prove fundamental theorem of algebra?

19. (a) Show that if $f(z)$ is analytic in a simply connected region Ω, then

$$\int_\gamma f(z)dz = 0$$

for every cycle γ in Ω?

    (b) Show that, if $pdx + qdy$ is locally exact in a region, then

$$\int_\gamma pdx + qdy = 0$$

for every cycle $\gamma \sim 0$ in Ω?

20. (a) State and prove Schwarz's Theorem?

    (b) Show that

$$\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}.$$

$10 \times 4 = 40$