P326 Reg.No:………………………… Name:……………………………………………………………………

MSc DEGREE EXAMINATION OCTOBER 2015
SEMESTER - 3, MATHEMATICS
COURSE CODE: P3MATT13- DIFFERENTIAL GEOMETRY

Time: 3 Hours **Max.Marks:75**

Part A

(Answer any 5 questions. Each carriers 2 marks)

1. Define the terms Graph and Level set of a function f: U→R; $U \subset R^{n+1}$
2. Find the Integral curve through (a b) for the vector field **X(p)=(**p, X(p)) where X(p)= -p
3. Define the Gauss map
4. Prove that the Covariant derivative is independent of the choice of orientation
5. Find ∇_v f where f = $2x_1^2+3x_2^2$; v= (1,0,2,1)
6. What do you mean by Curvature of a plane curve
7. Define Normal section of an n- surface at p determined by a unit vector **v**
8. State Inverse function theorem for an n-surface

Part B

(Answer any 5 questions .Each carriers 5 marks)

9. Let U be an open set in R^{n+1}. **X** be a smooth vector field on U. Suppose α: I → U is an integral curve of **X** with $\alpha(0)=\alpha(t_0)$ for some t_0 ε I; $t_0 \neq 0$, show that α is periodic
10. Let S be an n surface in R^{n+1}, S= $f^{-1}(c)$ where f: U→R is such that $\nabla f(q) \neq 0$ for all q ε S. Suppose g: U→R is a smooth function and p ε S is an extreme point of g on S ,then prove that there exists a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$
11. Prove that Geodesics have constant speed
12. Let S be an n- surface in R^{n+1}, let p, q ε S and let α be a piecewise smooth parameterized curve from p to q. Prove that $P\alpha : S_p \to S_q$ along α is linear and one to one
13. Compute the Weingarten map for the sphere $x_1^2+x_2^2+\ldots\ldots\ldots\ldots x_{n+1}^2 = r^2$; $r > 0$ oriented by **N= -** $\frac{\nabla f}{\|\nabla f\|}$
14. Prove that length of a parameterized curve is invariant under re parameterization