15. Let S be an oriented n- surface in R^{n+1} and let **v** be a unit vector in S_p ,$p \in S$. Prove that there exists an open set V subset of R^{n+1}containing p such that $S\cap \mathcal{N}(\boldsymbol{V})\cap V$ is a plane curve

16. Define a parameterized n-surface in R^{n+1} .Also give an example

Part C

(Answer either part a or part b .Each question carriers 10 marks)

17. (a) Let U be an open set in $R^{n+1}$and let $f : U \to R$ be smooth. Let $p \in U$ be a regular point of f, and let c = f(p). Prove that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $(\nabla f(p))^{\perp}$

    (b) Let $S \subset R^{n+1}$ be a connected n surface in R^{n+1}.Prove that there exist on S exactly two smooth unit normal vector fields $\mathbf{N_1}$ and $\mathbf{N_2}$ and $\mathbf{N_2}(p)= - \mathbf{N_1}(p)$ for all $p \in S$

18(a) Define spherical image of an oriented n surface. Also describe the spherical image of the cylinder $x_2^2+ \ldots\ldots\ldots\ldots x_{n+1}^2 = 1$ oriented by $\frac{\nabla f}{\|\nabla f\|}$ when n=1 and n=2

(b) Let S be a 2 surface in R^3 and let $\alpha : I \to S$ be a geodesic in S with $\dot{\alpha} \neq 0$.Prove that a vector field **X** tangent to S along α is parallel along α iff both $\|\mathbf{X}\|$ and the angle between **X** and $\dot{\alpha}$ are constant along α

19(a) Let C be a connected oriented plane curve and let $\beta: I \to C$ be a unit speed global Parameterization of C. Prove that β is either one to one or periodic

(b)Let S be the Ellipsoid $\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}+\frac{x_3^2}{c^2} = 1$, (a, b ,c $\neq 0$) oriented by the outward normal. Find the Gaussian curvature K (p)

20 (a) Let $\psi(\theta,\phi)= ((a+b \cos\phi)\cos\theta, (a+b \cos\phi)\sin\theta ,b \sin\phi)$be the parameterized torus in R^3.Find the Gaussian curvature

(b) Let V be a finite dimensional vector space with dot product and let $L: V\to V$ be a self ad joint linear transformation on V .Prove that there exists an orthonormal basis for V consisting of Eigen vectors of L