Scheme of Valuation

MSc DEGREE END SEMESTER EXAMINATION - OCTOBER 2016

SEMESTER -1: **MATHEMATICS**

COURSE: **16P1MATT02 - BASIC TOPOLOGY**

. Max. Marks :75

PART A

1. Definition of base on Page 92 — $\frac{1}{2}$ Mark
   Discrete topology — 1 Mark

2. Answer is "No" — $\frac{1}{2}$ Mark
   Any counter example — 1 Mark

3. Identifying the subspace topology as discrete topology — 1 Mark
   Concluding $\{0\}$ is open — $\frac{1}{2}$ Mark

4. Answer is "No" — $\frac{1}{2}$ Mark
   Example of an open set in Euclidean topology but not in semi-open interval topology — 1 Mark

5. Answer is "No" — $\frac{1}{2}$ Mark
   Any counter example — 1 Mark

6. Let Y be a subspace of X where X has a countable dense subset D. — $\frac{1}{2}$ Mark
   Proving $D \bigcap Y$ is dense in Y — 1 Mark

7. Identifying the statement is not true — $\frac{1}{2}$ Mark
   Any counterexample — 1 Mark

8. Counterexamples given on Page 151 — $1\frac{1}{2}$ Marks

9. Page 161 : Proposition 7.1.6 — $1\frac{1}{2}$ Marks

10. Suitable argument with co-countable topology — $1\frac{1}{2}$ Marks

PART B

11. Definition of open ball — $\frac{1}{2}$ Mark
    Definition of closed ball — $\frac{1}{2}$ Mark
    Identifying the statement is not true — 1 Mark
    Counterexample (may use discrete topology) — 3 Marks

12. Definition — 1 Mark
    Necessary part — 1 Mark
    Sufficiency Part — 3 Marks

13. Definition on Page 127 — 1 Mark
    Proof of Proposition 5.4.10 on Page 127 — 4 Marks

14. i. proof of compact in the cofinite topology — 3 Marks
    ii. $\{\{x\}; x \in \mathbb{R}\}$ is an open covering of $\mathbb{R}$ that has no finite subcover — 2 Marks

15. By looking at the image $\mathbb{Q}$ of $\mathbb{Q} \times \mathbb{Q}$ under projection map, we get $\mathbb{Q} \times \mathbb{Q}$ is disconnected — 2 Marks
    Proving complement is path connected and hence connected — 3 Marks

16. Proof of proposition 7.1.4 on Page 164 — 5 Marks

PART C

17. (a) Page:97-98 Proving Product topology is stronger — 5 Mark
    Converse — 5 Marks

    (b) Two definitions on Page 105 & 113 — 2 Marks
    Proof of Theorem 5.2.9 on page 113 — 8 Marks

18. (a) Define $F : x \to (x, f(x))$ which is clearly a bijection — 2 Marks
    Showing F is continuous — 6 Marks
    Proving F^{-1} exists and equals the restriction of projection — 2 Marks

    (b) Existence of weak topology Page : 125 — $4\frac{1}{2}$ Marks
    Definition of weak topology — $\frac{1}{2}$ Mark
    Topology generated by $\{(-\infty,\ 0), (-\infty,\ 0], (0,\ \infty), [0,\ \infty)\}$ — 5 Marks

19. (a) Proof of Theorem 6.2.6 on Page:144 — 10 Marks

    (b) Definition on Page 150 — 1 Mark
    Proof of Proposition 6.3.6 on page 152-153 — 9 Marks

20. (a) Proof of Theorem 7.1.15 on Page 164-165: Proving (i $\Rightarrow$ ii). — 4 Marks
    Proving (ii $\Rightarrow$ iii). — 3 Marks
    Proving (iii $\Rightarrow$ i). — 3 Marks

    (b) Proof of Theorem 7.2.8 on Page 170-171 — 10 Marks