Q. Code ··················  Name: ··················

Reg. No.: ··················

**MSc DEGREE END SEMESTER EXAMINATION - OCTOBER 2016**

SEMESTER -1: **MATHEMATICS**

COURSE: **16P1MATT02 - BASIC TOPOLOGY**

Time:3 hours  Max. Marks :75

## PART A

***Answer all questions. Each carries 1.5 marks.***

1. Define a base of a topological space $(X, \mathfrak{T})$. Describe the topology on $X$ having $\{\{x\}; x \in X\}$ as a basis.
2. Is $int(A \bigcup B) = int(A) \bigcup int(B)$ for sets $A, B$ in a topological space? Justify.
3. Suppose $\mathbb{Z}$ has the subspace topology induced by the Euclidean topology on $\mathbb{R}$. Is $\{0\}$ open in $\mathbb{Z}$? Justify.
4. Let $\mathfrak{T}_1, \mathfrak{T}_2$ denote the semi open interval topology and Euclidean topology on $\mathbb{R}$ respectively. Is the identity map $id : (\mathbb{R}, \mathfrak{T}_1) \to (\mathbb{R}, \mathfrak{T}_2)$ continuous? Justify.
5. Is every quotient map open? Justify.
6. Show that separability is preserved under continuous functions.
7. Prove or disprove that the interior of a connected set is connected
8. Give an example of a space which is connected but not locally connected.
9. Show that the limits of sequences are unique in a Hausdorff space.
10. Give an example of a space in which every compact subset is closed but which is not Hausdorff.

**$1.5 \times 10 = 15$**

## PART B

***Answer any 4 questions. Each carries 5 marks.***

11. Define an open ball and a closed ball in a metric space. Prove or disprove: In a metric space, a closed ball is the closure of the open ball with the same centre and radius.
12. Define a dense subset in a topological space $(X, \mathfrak{T})$. If $\mathfrak{B}$ is any base for $(X, \mathfrak{T})$, prove that a subset $D \subset X$ is dense in $X$ if and only $D$ intersects every non-empty member of $\mathfrak{B}$.

13. Define a quotient map. Show that every closed, surjective map is a quotient map.

14. Is $\mathbb{R}$ compact in the i) cofinite topology ii) discrete topology ?

15. Is $\mathbb{Q} \times \mathbb{Q}$ connected in $\mathbb{R}^2$ with the Euclidean topology? What about the complement of $\mathbb{Q} \times \mathbb{Q}$ in $\mathbb{R}^2$?

16. Show that every Tychonoff space is regular.

**$4 \times 5 = 20$**

## PART C

***Answer either (a) or (b) of the following 4 questions. Each carries 10 marks.***

17. (a) Show that the product topology and Euclidean topology on $\mathbb{R}^n$ coincide.

    (b) Define closure $\overline{A}$ and derived set $A'$ of a subset $A$ of a space $X$ and show that $\overline{A} = A \bigcup A'$.

18. (a) If $f : X \to Y$ is a continuous function, then show that $\{(x, f(x)); x \in X\}$ is homeomorphic to $X$.

    (b) Let $\{(Y_i, \mathfrak{T}_i); i \in I\}$ be a family of topological spaces and $f_i : X \to Y_i$ be functions for each $i \in I$. Define the weak topology determined by $f_i's$ and prove its existence. Describe the weak topology on the real line $\mathbb{R}$ generated by the single function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} -1 & \text{if } x < 0; \\ 0 & \text{if } x = 0; \\ 1 & \text{if } x > 0. \end{cases}$$

19. (a) Show that every closed and bounded interval is compact.

    (b) Define a locally connected space. Show that every quotient space of a locally connected space is locally connected.

20. (a) Show that the following statements are equivalent in a topological space $X$

    i. $X$ is regular

    ii. For any $x \in X$ and an open set $G$ containing $x$, there is an open set $H$ containing $x$ such that $\overline{H} \subset G$

    iii. The family of all closed neighborhoods of any point of $X$ forms a local base at that point.

    (b) Show that every regular, Lindelöff space is normal. **$10 \times 4 = 40$**