U130

SACRED HEART COLLEGE (AUTONOMOUS)

THEVARA, KOCHI -13

(Affiliated to Mahatma Gandhi University, Kottayam)

Reg. No. ....

Name..

B. Sc. DEGREE EXAMINATION-NOVEMBER 2014

FIRST SEMESTER - MATHEMATICS (COMPLEMENTARY)
COURSE-1: U1CPMAT1: DIFFERENTIAL CALCULUS AND TRIGONOMETRY

Time: Three Hours — Max. Marks:75

Part A(Short Answer Questions)

Answer all questions.
Each question carries 1 mark.

1. If $f(z) = 2.0001$, find $\lim_{z\to 2} f(z)$.
2. Give an example of a function whose first derivative and second derivative are the same.
3. State the product rule in differentiation.
4. Find the slope of the line, which is tangent to the curve $y = \sin^5 x$ at the point $x = \pi/3$.
5. State Rolle's Theorem.
6. Find the critical points of $4 - x^2$ in [-3,1].
7. Write the order and degree of $\frac{\partial^3 y}{\partial y^2 \partial x} = \left(\frac{\partial^2 y}{\partial x^2}\right)^2$
8. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ of $f(x, y) = (x^2 - 1)(y + 2)$.
9. Define hyperbolic cosine of x.
10. Separate sinh $(\alpha + i\beta)$ into real and imaginary parts.

(10 X 1 = 10)

Part B (Brief Answer Questions)

Answer any eight questions.
Each question carries 2 marks.

11. Find $\lim_{x\to 0} \dfrac{\tan 8x}{\sin 3x}$.
12. Find $\frac{dy}{dx}$ when $x = -\sqrt{u}$ and $y = u$.
13. Find $\frac{d^2y}{dx^2}$, when $y = \sec(\tan x)$.
14. Verify Lagrange's Mean Value Theorem for $x + \dfrac{1}{x}$, in $[\frac{1}{2}, 2]$.
15. Find the maximum and minimum values of $f(x) = |x - 5|$ in $4 \le x \le 7$.
16. Use chain rule to find $\frac{dw}{dt}$, where $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$.

17. Find all the second-order partial derivatives of $f(x,y) = \sin xy$.

18. Verify Laplace equation for $f(x,y,z) = 2z^3 - 3(x^2 + y^2)z$.

19. State expansions for $\sin x$ and $\cos x$ in terms of x.

20. Prove that $\sinh 3x = 3 \sinh x + 4 \sinh^3 x$.

(8 X 2 = 16)

Part C (Short Essay Type Questions)

Answer any five questions.
Each question carries 5marks.

21. Find dy/dt. where $y = 4\sin(\sqrt{1+\sqrt{t}})$.

22. Differentiate $\tan x$ w.r.t. x using first principle.

23. Find dy/dx, where $x + \sin y = xy$.

24. Find the absolute maximum and minimum values of $h(x) = \sqrt[3]{x}$ in the interval $-1 \leq x \leq 8$.

25. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at $(u, v) = (\frac{1}{2}, 1)$, where $w = xy + yz + xz$, $x = u + v$, $y = u - v$, $z = uv$.

26. Express $\sin^6 \theta$ in a series of cosines of multiples of θ.

27. Sum to n terms the series $\sin^3 \frac{\theta}{3} + 3\ \sin^3 \frac{\theta}{3^2} + 3^2\ \sin^3 \frac{\theta}{3^3} + \cdots$.

(5 X 5 = 25)

Part D (Essay)

Answer any two questions.
Each question carries 12marks.

28. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of $x = \cos t$, $y = \sqrt{3}\cos t$ at $t = \pi/3$.

29. Consider $f(x) = 3x^2 - 4x^3$
    (a) What are the critical points of f?
    (b) Find the intervals on which the function is increasing and decreasing.
    (c) Find the local minimum and maximum.

30. Find all the second order partial derivatives of the function: $f(x,y) = \dfrac{2x}{y + \sin x}$.

31. Separate into real and imaginary parts of $\sin^{-1}(\cos\theta + i\sin\theta)$.

(2 X 12 = 24)