U130

SACRED HEART COLLEGE (AUTONOMOUS)
THEVARA, KOCHI -13
(Affiliated to Mahatma Gandhi University, Kottayam)

Reg. No. ....

Name..

B. Sc. DEGREE EXAMINATION-NOVEMBER 2014
FIRST SEMESTER - MATHEMATICS (COMPLEMENTARY)
COURSE: U1CPMAT1: DIFFERENTIAL CALCULUS AND TRIGONOMETRY

Time: Three Hours

Max. Marks:75

Part A(Short Answer Questions)
Answer all questions.
Each question carries 1 mark.

1. Find $\lim\limits_{t \to 1} \dfrac{t^2 + t - 2}{t^2 - 1}$.
2. Give an example of a function whose first derivative and second derivative are the same.
3. State the quotient rule in differentiation.
4. Find $\frac{dy}{dx}$ when $x = a\cos t$ and $y = \sin t$.
5. State Lagrange's Mean Value Theorem.
6. Find the absolute minimum of $-(x+4)$ in [-4,1].
7. Write the order and degree of $\left(\frac{\partial y}{\partial x}\right)^2 = \frac{\partial^2 y}{\partial x^2}$
8. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ of $f(x, y) = \sqrt{x^2 + y^3}$.
9. When $x = \cos\theta + i\sin\theta$. prove that $x^n + \dfrac{1}{x^n} = 2\cos n\theta$.
10. Separate sinh $(\alpha + i\beta)$ into real and imaginary parts.

(10 X 1 = 10)

Part B (Brief Answer Questions)
Answer any eight questions.
Each question carries 2 marks.

11. Find $\lim\limits_{x \to \infty} \dfrac{3x^2 - 6x + 9}{6x^2}$.
12. Find the first and second derivatives of $y = \dfrac{3}{x^3} + \dfrac{2}{x^2} + \dfrac{4}{x}$.
13. Find an equation for the line perpendicular to the tangent to the curve $y = x^3 - 4x + 1$ at the point $(2, 1)$.
14. Verify Rolle's theorem for $\dfrac{x^3}{3} - 3x$. in $[-3, 3]$.

15. Find the extreme values of $f(x) = 2 - |x|$ in $-1 \leq t \leq 3$.

16. Find all the second-order partial derivatives of $f(x, y) = x + y - xy$.

17. Use chain rule to find $\frac{dw}{dt}$, where $w = x^2y^2$, $x = \cos t$, $y = e^{2t}$.

18. Verify Laplace equation for $f(x, y, z) = x^2 + y^2 - 2z^2$.

19. Show that $e^{-i\theta} = \cos\theta - i\sin\theta$.

20. Prove that $\cosh(\alpha + \beta) = \cosh\alpha \ \cosh\beta + \sinh\alpha \ \sinh\beta$.

(8 X 2 = 16)

Part C (Short Essay Type Questions)

Answer any five questions.

Each question carries 5marks.

21. Find dy/dt, where $y = \sqrt{1 - \sqrt{t}}$.

22. Differentiate $\operatorname{cosec} x$ w.r.t. x using first principle.

23. Find dy/dx, where $x + \tan(xy) = 0$.

24. Find the absolute maximum and minimum values of $h(x) = -\sqrt{5 - x^2}$ in the interval $-\sqrt{5} \leq x \leq 0$.

25. Find $\dfrac{\partial w}{\partial u}$ and $\dfrac{\partial w}{\partial v}$ at $(u, v) = (-2, 0)$, where $w = \log(x^2 + y^2)$, $x = u - v$ and $y = u + v$.

26. Express $\sin^7\theta$ in a series of sines of multiples of θ.

27. Separate into real and imaginary parts of $\tan^{-1}(\alpha + i\beta)$.

(5 X 5 = 25)

Part D (Essay)

Answer any two questions.

Each question carries 12marks.

28. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ of $x = \sec^2 t - 1$, $y = \tan t$ at $t = \pi/4$.

29. Consider $f(x) = 2x^2 - x^3$.

    (a) What are the critical points of f?

    (b) Find the intervals on which the function is increasing and decreasing.

    (c) Find the local minimum and maximum.

30. Find all the second order partial derivatives of the function: $f(x, y) = xe^y + x\cos y + y$.

31. Sum the series $1 + c \ \cos\alpha + c^2 \ \cos 2\alpha + \cdots\cdots$.

(2 X 12 = 24)