Q. Code: U223 Reg. No………………………… Name: ………………………………………………

B SC DEGREE END SEMESTER EXAMINATION APRIL 2015

SEMESTER - 2: SUBJECT- MATHEMATICS (CORE)

COURSE CODE: U2CRMAT2 - ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

Time: 3 Hrs. Maximum Marks: 75

PART A

Answer **all** questions. Each question carries **1** mark

1. Find the directrix of the parabola $(y+3)^2 = 2(x+2)$.
2. Define orthoptic locus of a conic.
3. Write the equation of the polar of (x_1, y_1) with respect to the parabola $y^2 = 4ax$.
4. Write the equation of the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
5. Write the polar equation of a circle whose centre is (c, α) and radius a.
6. Separate $\sin(\alpha - i\beta)$ into real and imaginary parts.
7. Write Gregory's series.
8. State De Moivre's theorem.
9. Find the eigen values of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.
10. Find two matrices of order 2 such that rank of A is 1, rank of B is 2 and rank of (A+B) is 2.

(10 x 1 = 10)

PART B

Answer any **eight** *questions. Each question carries* **2** *marks.*

11. Find the equation of the asymptotes of the hyperbola $6x^2$-7xy-3$y^2 + x + 4y = 0$.
12. Find the locus of the poles of chords of a parabola subtending a right angle at the vertex.

13. If $lx + my + n = 0$ is a normal to the parabola $y^2 = 4ax$, show that

$al^3 + 2alm^2 + m^2n = 0.$

14. Find the condition that the line $\frac{l}{r} = A\,cos\theta + Bsin\theta$ may be a tangent to the

conic $\frac{l}{r} = 1 + e\,cos\theta.$

15. If $u = \log\tan(\frac{\pi}{4} + \frac{\theta}{2})$, prove that $\cosh u = sec\theta$.

16. Show that $cosh^{-1}x = \log(x + \sqrt{x^2 - 1})$, if x is real.

17. Use De Moivre's theorem to solve the equation $x^4 - x^3 + x^2 - x + 1 = 0.$

18. Find the value of x using Cramer's rule from the following equations:

$3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4.$

19. Describe all elementary transformations of a matrix which do not change rank of a matrix.

20. Find the characteristic vectors of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

(8 x 2 = 16)

PART C

Answer any **five** *questions. Each question carries* **5** *marks.*

21. Find the locus of the point of intersection of perpendicular tangents of the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

22. Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from points on the circle

$x^2 + y^2 = c^2$. Show that the middle point of the points of contact lie on the

curve $(\frac{x^2}{a^2} + \frac{y^2}{b^2})^2 = \frac{x^2+y^2}{c^2}$.

23. A circle passing through the focus of a conic whose latus rectum is $2l$ meets

the conic in four points whose distances from the focus are r_1, r_2, r_3, r_4.

Prove that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{l}$.

24. Sum to n terms and to infinity the series

$1 + a\,cos\theta + a^2 \cos 2\theta + a^3 \cos 3\,\theta + \cdots$ ... . where $|a|<1$.

25. Separate into real and imaginary parts of the expression $tan^{-1}(x + iy)$.

26. Solve completely the system of equations : $x - 2y + z - w = 0$,
$x + y - 2z + 3w = 0,\ \ 4x + y - 5z + 8w = 0,\ \ 5x - 7y + 2z - w = 0$.

27. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(5 x 5 = 25)

PART D

Answer any **two** *questions. Each question carries* **12** *marks.*

28. (a) Prove that the tangent at the extremities of any focal chord of a parabola intersect at right angles on the directrix.

(b) Show that the eccentric angles of the extremities of a pair of conjugate diameters of an ellipse differ by a right angle.

29. If PSP' is a focal chord of a conic, show that the angle between the tangents at P and P' is $tan^{-1}\left(\frac{2esin\alpha}{1-e^2}\right)$, where α is the angle between the chord and the major axis.

30. Factorise $x^7 - 1$ into real factors.

31. Find non singular matrices P and Q such that PAQ is in the normal form for the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

(2 x 12 = 24)