Q.Code:U315 Reg. No:...................... Name:.............................

B.Sc DEGREE END SEMESTER EXAMINATION OCTOBER 2015

Semester-3: CORE COURSE FOR B.Sc MATHEMATICS/ COMPUTER APPLICATIONS
COURSE: U3CRMAT3- U3CRCMT3- CALCULUS

Time: 3 Hrs. Maximum Marks:75

Part A

Answer all questions.
Each question carries 1 mark.

1. If $y = e^{mx}$, find $\frac{d^n y}{dx^n}$.
2. State Maclaurin's Theorem.
3. Define point of inflexion.
4. For $f(x, y) = x^2 + 3xy + y - 1$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(4, -5)$.
5. Find the Hessian Matrix of $f(x, y) = xy$.
6. Evaluate $\int\limits_0^1 r\sqrt{1-r^2}dr$.
7. State Cavalier's Principle.
8. Write the formula for the length of a smooth curve with parametrisation $x = f(t)$, $y = g(t)$; $a \le x \le b$.
9. Write down the coordinate conversion formula from spherical coordinates to rectangular coordinates.
10. If $x = r\cos\theta$ and $y = r\sin\theta$, then find the Jacobian $J(r, \theta)$.

Part B

Answer any eight questions.
Each question carries 2 marks.

11. Find the n^{th} derivative of $\cos^4 x$.
12. Show that the curve $ay^2 = x(x-a)(x-b)$ has two and only two points of inflexion.
13. If $f(x, y) = \ln\sqrt{x^2 + y^2}$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
14. Evaluate $\frac{dw}{dt}$ if $w = 2ye^x - \ln z$, $x = \ln(t^2 + 1)$, $y = \tan^{-1} t$, $z = e^t$.
15. Find the local extreme values of the function $f(x, y) = xy$.
16. Find the area of the region enclosed by $y = x^2 - 2x$ and $y = x$.
17. A solid lie between planes perpendicular to the x-axis at $x = -1$ and $x = 1$ where the cross-sections perpendicular to the x-axis are squares whose diagonals run from the semicircle $y = -\sqrt{1-x^2}$ to the semicircle $y = \sqrt{1-x^2}$. Find the volume of the solid.
18. Evaluate $\int\limits_0^{\pi}\int\limits_0^{\sin x} y\,dy\,dx$.

19. Change the cartesian integral into an equivalent polar integral and evaluate the integral $\int_0^6 \int_0^y x dx dy$.

20. Find a spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$.

Part C

Answer any five questions.
Each question carries 5 marks.

21. For the cycloid $x = a(t + \sin t)$, $y = a(1 - cost)$, prove that $\rho = 4a \cos \frac{t}{2}$.

22. Show that the evolute of the ellipse $x = a \cos\theta$, $y = b \sin\theta$ is $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$.

23. If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

24. State Pappu's Theorem for volumes. Using it find the volume of the torus generated by revolving a circular disk of radius a about an axis in its plane at a distance $b \geq a$ from its center.

25. The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ is revolved about the y-axis to generate a solid. Find the volume of the solid using cylindrical shell method.

26. Evaluate $\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2} dx dy$ by applying the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$.

27. $\int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^1 \rho^4 \sin^3(\phi) d\rho d\phi d\theta$.

Part D

Answer any two questions.
Each question carries 12 marks.

28. Show that the n^{th} derivative of $y = \tan^{-1}(\frac{1+x}{1-x})$ is $(-1)^{n-1}(n-1)! \sin^n\theta \sin n\theta$ where $\theta = \cot^{-1} x$.

29. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

30. (a) Find the length of the curve $y = (\frac{x}{2})^{\frac{3}{2}}$ from $x = 0$ to $x = 2$.

    (b) Find the area of the surface generated by revolving the curve $y = \sqrt{2x - x^2}$; $\frac{1}{2} \leq x \leq \frac{3}{2}$ about the x-axis.

31. (a) Reverse the order of integration and evaluate the integral $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dy dx$.

    (b) Find the average value of $f(x, y) = \cos(xy)$ over the rectangle $R : 0 \leq x \leq \pi$, $0 \leq y \leq 1$.

    (c) Find the volume of the tetrahedron cut from the first octant by the plane $6x+3y+2z = 6$.