Reg. No……………………………………………. Name…………………………………………….

B. Sc. DEGREE END SEMESTER EXAMINATION MARCH 2018

SEMESTER – **4: MATHEMATICS (CORE COURSE FOR MATHEMATICS & COMPUTER APPLICATION)**

COURSE: **15U4CRMAT4-15U4CRCMT04: VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS**

Common for Regular (2016 Admission) & Supplementary (2015 & 2014 Admissions)

Time: Three Hours Max. Marks: 75

PART A

*Answer **all** questions. Each question carries **1** mark*

1. Find an equation for the plane through P(-3, 0, 7) perpendicular to $n = 5i + 2j - k$
2. Define Radius of curvature of a curve at any point P
3. The position vector of a particle in space at time t is $r(t) = (t+1)i + (t^2 - 1)j + 2tk$. Find the particles direction of motion at $t = 1$
4. State Gauss-Divergence theorem
5. Find the parameterization of the cone $z = \sqrt{x^2 + y^2}, \quad 0 \le z \le 1$
6. Show that $Curl(\frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k) = 0$
7. Define standard reciprocal equation
8. If α, β, γ are the roots of the equation $x^3 + 3x + 2x + 1 = 0$. The value of $\sum \frac{1}{\alpha\beta}$
9. Write the formula to compute the percentage error
10. Find the function $\emptyset(x)$ for which the iteration method can be applied to find a real root of $x^3 + x^2 - 1 = 0$ (1 x 10 = 10)

PART B

*Answer **any eight** questions. Each question carries **2** marks*

11. Find the distance from S(1, 1, 3) to the plane 3x + 2y + 6z = 6
12. Find the derivative of $f(x, y, z) = xy + yz + zx$ at $P(1, -1, 2)$ in the direction of $u = 3i + 6j - 2k$
13. Find the unit tangent vector of the curve $r(t) = (tsint + cost)i + (sint - tcost)j$; $\sqrt{2} \le t \le 2$
14. Show that $F = (ysinz)i + (xsinz)j + (xycosz)k$ is conservative
15. Evaluate $\int_c \; x + y + z \, ds$ over the straight line segment from (1, 2, 3) to (0, -1, 1)
16. Find the work done by the force field $F = xi + yj + zk$ in moving an object along the curve C parameterized by $r(t) = \; Cos(\pi t)i + t^2 j + \sin(\pi t)\, k, \; 0 \le t \le 1$
17. Solve $6x^3 - 11x^2 - 3x + 2 = 0$, given that the roots are in Harmonic Progression
18. Find the equation whose roots are the roots of the equations $2x^5 - 9x^3 + 4x + 3 = 0$ each increased by 2

19. Find a root of the equation $x^3 - 9x + 1 = 0$ correct to two decimal places using bisection method
20. Use the method of iteration to find a positive root between 0 and 1, of the equation $xe^x = 1$ correct to two decimal places (2 x 8 = 16)

PART C

*Answer **any Five** questions. Each Question carries **5** marks*

21. Find the point on the curve $r(t) = (5sint)i + (5cost)j + 12tk$ at a distance 26π units along the curve from the origin in the direction of increasing arc length
22. Find the principal unit normal vector N for the helix $r(t) = (acost)i + (asint)j + btk, \quad a, b \geq 0,\ a^2 + b^2 \neq 0$
23. Show that the differential form $x^2dx + yzdy + \left(y^2/_2\right)dz$ is exact and evaluate the integral

$$\int_{(0,0,0)}^{(0,3,4)} x^2dx + yzdy + \left(y^2/_2\right)dz$$

24. State Green's theorem. Using Green's theorem evaluate $\oint_c \ y^2dx + x^2dy$, where C is the triangle bounded by $x = 0,\ x + y = 1,\ y = 0$
25. Solve the equation $x^5 - 3x^4 - 14x^3 - 14x^2 - 3x + 1 = 0$
26. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $\beta + \gamma - 2\alpha,\ \gamma + \alpha - 2\beta,\ \alpha + \beta - 2\gamma$
27. Find a root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places using Method of false position (5 x 5 = 25)

PART D

*Answer **any Two** questions. Each question carries **12** marks*

28. a) Find the Binormal vector B for the space curve $r(t) = (e^t cost)i + (e^t sint)j + \sqrt{2}e^t k$ at $t = 0$

    b) Find parametric equations for the line tangent to the curve of intersection of the surfaces $f(x, y, z) = xyz - 1 \ and\ g(x, y, z) = x^2 + 2y^2 + 3z^2 - 6$ at the point $P_0(1, 1, 1)$
29. State Gauss Divergence Theorem. Verify Gauss Divergence theorem for the field $F = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.
30. a) Solve $x^3 - 6x^2 + 3x - 2 = 0$ using Cardon's Method

    b) Solve $x^4 - 2x^3 - 3x^2 + 4x - 1 = 0$ given that the product of two roots is unity
31. Explain Newton – Raphson Method. Using Newton Raphson Method find a root of the equation $xsinx + cosx = 0$ correct upto four decimal places (12 x 2 = 24)

****