Reg. No………………………………………………. Name…………………………………………… **18U339**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2018

SEMESTER **– 3: MATHEMATICS (COMPLEMENTARY COURSE FOR B SC PHYSICS & CHEMISTRY)**

COURSE: **15U3CPMAT3 – VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY**

(For Regular 2017 Admission and Supplementary / Improvement 2016, 2015 & 2014 Admission)

Time: Three Hours Max. Marks: 75

Part A

Each Question carries 1 Mark Answer All Questions

1. The curvature of a circle of radius a is …………………
2. The rule for finding $\nabla(fg)$ is ……………………..
3. Define the term potential function for a field.
4. State Stokes' theorem.
5. Define divergence of a vector field.
6. Solve $(1 + \cos\theta)dr = r\sin\theta d\theta$.
7. $f(x,y) = x^4 \sin(\frac{x}{y})$ is a homogeneous function of degree …………………..
8. Give the definition of an exact differential equation.
9. Find the focus of the parabola $y^2 = 10x$.
10. Find the polar equation of the circle with center (3, 0) and radius 3.

(1 x 10 = 10)

Part B

Each Question carries 2 Marks answer any Eight.

11. Find $\int (\cos t\, i + j - 2t\, k)\, dt$.
12. Find the gradient of the function $f(x,y) = y - x$ at (2,1).
13. Evaluate $\int_C (x + y)$, where C is the straight-line segment $x = t$, $y = (1 - t)$, $z = 0$, from (0, 1, 0) to (1, 0, 0).
14. Check whether the field $F = (z + y)\, i + z\, j + (y + x)\, k$ is conservative or not.
15. Show that the differential form in the integral $\int_{(1,1,2)}^{(3,5,0)} yz\, dx + xz\, dy + xy\, dz$ is exact.
16. Find the directrix of the parabola $r = \frac{25}{10 + 10\cos\theta}$.
17. Draw the graph of the polar coordinates satisfying $1 \le r \le 2$ and $0 \le \theta \le \pi/2$.
18. Give the order and degree of the differential equation $xy\left(\frac{dy}{dx}\right)^2 + (x^2+y^2)\frac{dy}{dx} + xy = 0$.
19. Solve $\frac{dy}{dx} = -\frac{2x+3y}{3x+2y}$.
20. Find the slope and intercept of $r\cos\theta + r\sin\theta = 1$ by finding its equivalent Cartesian equation.

(2 x 8 = 16)

Part C

Each Question carries 5 Marks Answer Any Five.

21. Find the principal unit normal vector N and unit tangent vector T for the circular motion $r(t)$ = $(\cos 2t)\, i + (\sin 2t)\, j$.
22. Find the directions in which the derivative of the function $f(x,y) = xy + y^2$ at $P(3,2)$ equal to zero.
23. A fluids velocity field is $F = x\, i + z\, j + y\, k$. Find the flow along the helix $r(t) = (\cos t)i + (\sin t)\, j + t\, k$, $0 \leq t \leq \pi/2$.
24. Let F be a differentiable vector field and let $g(x,y,z)$ be a differentiable scalar function. Show that $\nabla \cdot (gF) = g\nabla \cdot F + \nabla g \cdot F$.
25. Solve the differential equation $y^2 - 1 - p^2 = 0$ for y, where $p = \frac{dy}{dx}$.
26. Check for exactness and solve $xydx + (2x^2 + 3y^2 - 20)dy = 0$.
27. Find the eccentricity, foci and directrices of the ellipse $6x^2 + +9y^2 = 54$.

(5 x 5 = 25)

Part D

Each Question carries 12 Marks Answer Any Two.

28. Use the Divergence Theorem to find the outward flux of $F = y\, i + xy\, j - z\, k$ across the boundary of the region D, where D is the region inside the solid cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$.
29. Use Green's Theorem to find the counterclockwise circulation and outward flux for the field F = $(y^2 - x^2)\, i + (x^2 + y^2)\, j$ and the curve C, where C is the triangle bounded by $y = 0, x = 3$ and $y = x$.
30. Give the standard form of a Bernoulli's differential equation. Solve $\frac{dy}{dx} + \frac{1}{x}y = 3y^3$.
31. Identify the conic $x^2 - y^2 - 2x + 4y = 4$. Find its center, foci, vertices and asymptotes.

(12 x 2 = 24)

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