Reg. No.. Name.. **18U224**

B. Sc. DEGREE END SEMESTER EXAMINATION MARCH / APRIL 2018

SEMESTER **– 2: B. Sc. MATHEMATICS (CORE COURSE)**

COURSE: **15U2CRMAT2 – ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES**

(Common for Regular 2017 / Supplementary - Improvement 2016 / 2015 / 2014 Admission)

Time: Three Hours Max. Marks: 75

PART A

*Answer **all** questions. Each question carries **1** mark.*

1. Define polar of a point with regard to a circle.
2. What is the parametric representation of a point on $y^2 = 4ax$.
3. Define the term latus rectum.
4. What are conjugate lines?
5. What are the asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
6. Find the imaginary part of sin (α + βi).
7. What is the centre of the circle $r = asin\Theta$.
8. Prove that $\sin(i\Theta) = i\sinh\ \Theta$.
9. If A is a 3 × 3 matrix of rank 3, what can you say about the solutions of the linear system AX = 0?
10. Le A be a 2×2 matrix with Eigen values 1 and -1. Find the determinant of A^{100}.

(1 × 10 = 10)

PART B

*Answer **any eight** questions. Each question carries **2** marks.*

11. Prove that the pole of the line $lx + my + n = 0$ with respect to $x^2 + y^2 = a^2$ is $(-\frac{la^2}{n}, -\frac{ma^2}{n})$.
12. Find the conditions for the lines $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$ are conjugate with respect to $x^2 + y^2 = a^2$.
13. Find the polar of a point (x_1, y_1) with respect to the parabola $y^2 = 4ax$.
14. Find the condition in order that the line $\frac{l}{r} = Acos\Theta + Bsin\Theta$ may be a tangent to the conic $\frac{l}{r} = 1 + ecos\Theta$.
15. Trace the curve $r = acos\Theta, 0 \leq \Theta \leq \pi$.
16. Find the equation of the chord joining two points t_1 and t_2 on $y^2 = 4ax$.
17. Express $\tan(\alpha + \beta i)$ in the form $A+Bi$.
18. Prove that $cosh^{-1} x = \log(x + \sqrt{x^2 - 1})$.
19. Using Cramer's rule solve: $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$.
20. If λ is a nonzero Eigen value of A, prove that $\frac{1}{\lambda}$ is an Eigen value of A^{-1}.

(2 × 8 = 16)

PART C

*Answer **any five** questions. Each question carries **5** marks.*

21. Derive the condition for two circles to cut one another orthogonally.
22. Find the equation of the normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at a point (x_1, y_1).
23. Obtain the equation of a rectangular hyperbola referred to its asymptotes as axes.
24. Sum the series $\sin^3\left(\frac{\theta}{3}\right) + 3\sin^3\left(\frac{\theta}{3^2}\right) + 3^2\sin^3\left(\frac{\theta}{3^3}\right) + \ldots..$
25. Find the factorization of $x^n - 1$ when n is even.
26. Reduce the following matrix to its normal form and hence find its rank. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$.
27. Find all solutions of the system : $x + 2y + 3z = 0, 3x + 4y + 4z = 0, 7x + 10y + 12z = 0.$

(5 × 5 = 25)

PART D

*Answer **any two** questions. Each question carries **12** marks*

28. Find the equation of the pair of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the point (x_1, y_1).
29. If two conics have a common focus, show that two of their common chords pass through the point of intersection of their directrices.
30. Prove that $\tan\theta = 8\theta\left[\frac{1}{\pi^2 - 4\theta^2} + \frac{1}{3^2\pi^2 - 4\theta^2} + \frac{1}{5^2\pi^2 - 4\theta^2} + \cdots\right]$.
31. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$. Show that the equation is satisfied by A and hence find A^{-1}.

(12 × 2 = 24)

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