Reg. No…………………………………………. Name…………………………………………… **18U141**

B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER/NOVEMBER 2018

SEMESTER **–1: MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS/CHEMISTRY)**

COURSE: **15U1CPMAT1: DIFFERENTIAL CALCULUS AND TRIGONOMETRY**

(Common for Regular 2018 admission and improvement 2017/ supplementary 2017/2016/2015/2014 admission)

Time: Three Hours Max. Marks: 75

PART A (Short Answer Questions)

*Answer **all** questions. Each question carries **1** mark.*

1. Evaluate $\lim_{\theta \to 0} \frac{\sin 3\theta}{4\theta}$
2. Evaluate $\lim_{x \to \infty} \frac{2x+3}{5x+7}$
3. State Mean Value Theorem.
4. Define increasing function.
5. Find $\frac{\partial f}{\partial x}$, when $f(x, y) = e^{xy}$
6. State mixed derivative theorem for partial derivatives.
7. What is the imaginary part of $\sin(x - iy)$.
8. Define hyperbolic cosine of x.
9. State De-Moivre's theorem.
10. What is the period of sinhx. (1 x 10 = 10)

PART B (Brief Answer Questions)

*Answer **any eight** questions. Each question carries **2** marks.*

11. Differentiate $y = (x^2 + 1)^8$ with respect to x.
12. If $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \to 0} f(x)$.
13. Using Mean value theorem, find a constant 'c' for the function $f(x) = x^2 + 2x - 1 \; on \; [0,1]$.
14. Find the absolute maximum of the function $f(x) = 8x - x^4$ on [-3, 1].
15. Verify Rolle's theorem for the function $f(x) = x^3 - 9x$.
16. Draw a tree diagram and write a chain rule for $\frac{\partial y}{\partial r}$ for $y = f(u), u = g(r, s)$
17. Find $\frac{\partial f}{\partial x} \; and \; \frac{\partial f}{\partial y}$ when $f(x, y) = \tan^{-1}({}^{y}/_{x})$.
18. Separate $\sinh(x + y)$ into real and imaginary parts.
19. Prove that $\cosh^{-1} x = \log[x + \sqrt{x^2 - 1}]$
20. Prove that $cosh^2 x - sinh^2 x = 1$. (2 x 8 = 16)

PART C (Short Essay Type Questions)

*Answer **any five** questions. Each question carries **5** marks*

21. Find

$(a) \lim_{x \to \infty} xsin({}^{1}/_{x})$ $(b) \lim_{\theta \to 0} \frac{1 - \cos 3\theta}{\theta^2}$.

22. Find an equation for the tangent to the curve $y = x + \frac{2}{x}$ at (1, 3).
23. Find the critical points of $f(x) = (-x^3 + 12x + 5)\ on -3 \leq x \leq 3$. Identify intervals on which the function is increasing or decreasing. Find function local and absolute extreme values.
24. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s, where $w = x^2 + y^2, x = r - s, y = r + s$.
25. If $z = \log\sqrt{x^2 + y^2}$. Prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$
26. Prove that $\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$.
27. If $\sin(A + B) = x + iy$. Prove that $(a)\ \frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ $\quad (b)\ \frac{x^2}{\sinh^2 A} - \frac{y^2}{\cosh^2 A} = 1$

(5 x 5 = 25)

PART D (Essay)

*Answer **any two** questions. Each question carries **12** marks.*

28. (a) Let $f(x) = x + 1$ and $\in = 0.01$, find a $\delta > 0$ such that $\forall x$ with $0 < |x - 4| < \delta$, the inequality $|f(x) - 5| < \in$ holds .

    (b) Find the horizontal asymptote of the curve $f(x) = \frac{5x^2 + 8x - 3}{3x^2 + 2}$.
29. (a) Find the normal line to the curve $x^3 + y^3 - 9xy = 0\ at\ (2, 4)$.

    (b) Find $\frac{d^2y}{dx^2}$ if $ax^2 + 2hxy + by^2 = 1$ where a, b, h are constants.
30. (a) Using chain rule, evaluate $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2; x = \frac{r}{s}$, $y = r^2 + \log s, z = 2r$.

    (b) Find all second order partial derivatives of the function $f(x, y) = x \sin y + y$.
31. (a) Separate $\tan^{-1}(x + iy)$ into real and imaginary part.

    (b) Sum the series $\frac{c\sin\theta}{1!} + \frac{c^3 \sin 3\theta}{3!} + \frac{c^5 \sin 5\theta}{5!} + \cdots$

(12 x 2 = 24)

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