

B.Sc./B.C.A DEGREE END SEMESTER EXAMINATION - OCTOBER/NOVEMBER 2018

SEMESTER – 1: MATHEMATICS (COMMON FOR MATHEMATICS, COMPUTER APPL. & B. C. A.)

COURSE: 15U1CRMAT1-15U1CRCMT1-16U1CPCMT1: FOUNDATION OF MATHEMATICS

(Common for Regular 2018 admission and improvement 2017/supplementary 2017/2016/2015/2014 admission)

Time: Three Hours

Max. Marks: 75

PART A (Short Answer Questions)*Answer all questions. Each question carries 1 mark.*

1. Let A, B and C be 3 sets. Prove that $A \cup (B \cup C) = (A \cup B) \cup C$.
2. Find the domain and range of the following function $f(x) = \frac{1}{x-2}$.
3. Find the power set of $\{\emptyset, \{a\}, \{\emptyset, a\}\}$?
4. Find the lexicographic ordering of the pair (1, 1, 2) and (1, 2, 1).
5. Define n-ARY relations.
6. Define contrapositive proposition.
7. Write the negation of "The summer in Marine is hot and sunny"
8. Define Euler's Function.
9. State Unique Factorization Theorem.
10. Find the remainder when 8^{30} is divided by 31. (1 x 10 = 10)

PART B (Brief Answer Questions)*Answer any eight questions. Each question carries 2 marks.*

11. Show that the set of all integers is countable.
12. Obtain the inverse of the function $f(x) = \left\{ \left(x, \frac{x}{x-1} \right), -1 \leq x \leq 1 \right\}$.
13. Define POSET. Give an example of a POSET that has a least member but no greatest members.
14. Define linearly ordered set with example.
15. Let $S = \{1,2,3,4,5\}$ and R the relation, $R = \{(1,3),(2,4),(3,5),(1,1),(2,2),(4,2),(3,1)\}$. Is this relation on S an equivalence relation? Justify.
16. Use De Morgan's law to find the negation of " Paris is in France and London is in England"
17. Prove that $\sim \forall x P(x) \equiv \exists x \sim P(x)$.
18. Find the highest power of 5 contained in 158!
19. Prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7.
20. Solve $3x \equiv 5 \pmod{11}$. (2 x 8 = 16)

PART C (Short Essay Type Questions)*Answer any five questions. Each question carries 5 marks.*

21. Compute

(i)
$$\sum_{k=1}^n \frac{1}{k(k+1)}$$

(ii)
$$\sum_{k=50}^{100} k^2$$

22. Let R_1 and R_2 be the "congruent modulo 3" and congruent modulo 4" relations respectively on the set of integers. Find

- (i) $R_1 \cup R_2$
- (ii) $R_1 \cap R_2$
- (iii) $R_1 - R_2$
- (iv) $R_1 \oplus R_2$.

23. Define equivalence relation. Let \sim be an equivalence relation on A , then show that for any $a, b \in A$

- (i) $a \in [a]$
- (ii) $[a] \cap [b] = \emptyset$ or $[a] = [b]$
- (iii) A is the union of all disjoint classes.

24. Use quantifier to express the statement that "There exist a woman who has taken a flight on every airline in the world".

25. For each of these statement find a domain for which the statement is true and a domain for which the statement is false.

- (i) Everyone speaks Hindi.
- (ii) There is someone other than 21 years.
- (iii) Every two people have the same first name.
- (iv) Someone knows more than two other people.
- (v) Everyone is studying discrete mathematics.

26. If n is a prime number greater than 7, show that $n^6 - 1$ is divisible by 504.

27. Find the highest power of 7 contained in $1000!$ (5 x 5 = 25)

PART D (Essay)

Answer **any two** questions. Each question carries **12** marks.

28. Prove that

- (i) $(A - B) - C = (A - C) - (B - C)$
- (ii) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

29.

- (i) Determine the relation represented by the matrix $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ is a partial order.

- (ii) Let \mathbf{R} be the relation on the set of real numbers such that $a\mathbf{R}b$ iff $a-b$ is an integer. Prove that \mathbf{R} is an equivalence relation.

30. Test the validity of the following: If Ashok wins then Ram will be happy. If Kamal wins, Raju will be happy. Either Ashok will win or Kamal will win. However if Ashok wins, Raju will not be happy and if Kamal wins Ram will not be happy. So Ram will be happy if and only if, Raju is not happy.

31. (i) State and prove that Wilson's Theorem.

- (iii) State and prove Euler's Extension of Fermat's theorem.

(12 x 2 = 24)