Reg. No .... Name 18P3631

MSc DEGREE END SEMESTER EXAMINATION - OCTOBER 2018

SEMESTER 3 : MATHEMATICS

COURSE : **16P3MATT13 : GRAPH THEORY**

(For Regular - 2017 Admission & Supplementary - 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A

Answer the following (1.5 marks each)

1. Define a highly irregular graph. Give an example. Show that there is no connected highly irregular graph of order 3.
2. Define edge cut and cut edge and illustrate with examples.
3. Give an example of a tree with disjoint center and centroid.
4. Define branch at a vertex u of a tree T.Define weight of a vertex u of a tree T.Also define the centroid of a tree T.
5. Prove or disprove: If closure of G is Hamiltonian, then G is Hamiltonian.
6. Define proper vertex coloring and chromatic number of a graph G.
7. Define maximum matching and maximal matching in a graph. Give an example of a maximal matching which is not a maximum matching.
8. Give an example of a cubic graph with edge chromatic number 4.
9. Show that a graph is planar if and only if it is embeddable on a sphere.
10. Let G be a plane graph of order n and size m in which every face is bounded by a k-cycle. Show that m = k(n − 1)/(k − 2).

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Explain the concept of isomorphism of graphs. Illustrate the concept by giving a pair of graphs that are isomorphic and a pair of graphs that are not isomorphic.
12. (a) Show that a tree with at least two vertices contains at least two pendant vertices.
    (b) Show that if $\delta(G) \geq 2$, then G contains a cycle.
13. What is the total number of branches in a Tree with n vertices.
14. Prove that a simple k-regular graph on $2k-1$ vertices is Hamiltonian.
15. Prove that for any graph G, $\alpha(G)+\beta(G)=n$
16. Briefly describe the Four-Color Theorem.

(5 x 4 = 20)

Section C

Answer the following (10 marks each)

17.1. State and prove Whitney's theorem on 2- connected graphs.

OR

2. Show that a diconnected tournament with $n \geq 3$ is vertex pancyclic.

18.1. (a) Show that every tree has a center consisting of either a single vertex or two adjacent vertices.

(b) If e is not a loop of G, show that $\tau(G) = \tau(G - e) + \tau(G.e)$.

OR

2. Find the number of spanning trees of the graph $C_3 \vee K_1$

19.1. Show that a graph G is Eulerian if and only if it has an odd number of cycle decompositions.

OR

2. Show that for every positive integer k, there exists a triangle-free graph with chromatic number k.

20.1. Show that a graph G is planar if and only if each of its blocks is planar.

OR

2. Show that $\chi'(K_n) = \begin{cases} n-1 & \text{if } n \text{ is even,} \\ n & \text{if } n \text{ is odd.} \end{cases}$

(10 x 4 = 40)