Reg. No ..... Name **18P3604**

MSc DEGREE END SEMESTER EXAMINATION - OCTOBER 2018

SEMESTER 3 : MATHEMATICS

COURSE : **16P3MATT11 : PARTIAL DIFFERENTIAL EQUATIONS**

(For Regular - 2017 Admission & Supplementary - 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A
Answer any 10 (1.5 marks each)

1. Define Pfaffian differential equation with example
2. Verify that the differential equation $y\,dx + x\,dy + 2z\,dz = 0$ is integrable
3. Derive a partial differential equation from $x^2 + y^2 = (z-c)^2 \tan^2\alpha$
4. Prove that $z = \sqrt{2x+a} + \sqrt{2y+b}$ is a complete integral of the pde $z = \frac{1}{p} + \frac{1}{q}$
5. Find the complete integral of the equation $pqz = p^2(xq + p^2) + q^2(yp + q^2)$
6. Define parabolic, elliptic, hyperbolic equations
7. Solve

$$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = \frac{2\partial^4 z}{\partial x^2 \partial y^2}$$

8. Find the particular integral of

$$r + 3s + 2t = x + y$$

9. Write the Laplace's equation
10. State exterior Neumann problem

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Find the integral curves of $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$
12. Find the orthogonal trajectory on the surface $x^2 + y^2 + 2fyz + d = 0$ of its intersection with the family of planes parallel to xy plane
13. Show that the equation $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve
14. Solve

$$(2D - D' + 4)(D + 2D' + 1)Z = 0$$

15. Solve

$$(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = e^{x+y}$$

16. Show that if a function z satisfies the differential equation $\frac{\partial^2 z}{\partial x^2}\frac{\partial z}{\partial y} = \frac{\partial^2 z}{\partial x \partial y}\frac{\partial z}{\partial x}$ it is of the form $f(x + g(y))$, where f and g are arbitrary

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. Find the integral surface of pde $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$ through the curve $xz = a^3,\ y = 0$

OR

2. Find the integral surface of pde $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0,\ x^2 + y^2 = 2x$

18.1. (i) Derive the condition for compatibility of system of first order partial differential equations
(ii) Show that the equation $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve

OR

2. Find the complete integral of the $p^2x + qy = z$ and hence derive the equation of the integral surface of which the line $y = 1, x + z = 0$ is a generator.

19.1. (i) Solve $(D^2 - D'^2)z = x - y$.
(ii) Solve $(2D - D' + 4)(D + 2D' + 1)z = 0$.

OR

2. Deduce the equation

$$y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$$

to canonical form and hence solve it.

20.1. Describe Monge's method. Solve the equation $r = t$ the wave equation using Monge's method

OR

2. Describe Monge's method. Solve
$z^2(rt - s^2) + z(1 + q^2)r - 2pqzs + z(1 + p^2)t + 1 + p^2 + q^2 = 0$ using Monge's method

(10 x 4 = 40)