Reg. No .... Name **18P440**

M Sc DEGREE END SEMESTER EXAMINATION - APRIL 2018

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT19EL ; THEORY OF WAVELETS**

(For Regular - 2016 admission)

Time : Three Hours Max. Marks: 75

Section A
Answer all the following (1.5 marks each)

1. Write an orthonormal basis for $l^2(Z_2)$ other than the standard basis.
2. Prove that $\hat{z}(m) = \sqrt{N} < z, E_m >; 0 \leq m \leq N-1$.
3. If $z = (2, 5, -1, i)$, find U(z). Hence verify that D o U(z)=z. Also verify that $UoD(z) = \frac{1}{2}(z + z^*)$.
4. If N is divisible by 2^l, define $D^l : l^2(Z_N) -> l^2(Z^l_{N/2})$.
5. With the usual notations express f_l and g_l interms of upsampling operators and convolutions.
6. Suppose X is an innerproduct space with subspaces U,V and W. Suppose $U \perp V$ and $U \perp W$ and $U \oplus V = U \oplus W$. Prove that V=W.
7. Suppose A=$\{a_j\}_{j \in Z}$ is an orthonormal set in a Hilbert space H. Then define S_A.
8. Define the trigonometric system. Hence define a trigonometric polynomial. Is $sin(\theta - \theta_0)$ a trigonometric polynomial ? justify.
9. For $z, w \in l^2(Z)$, define $z * w$.
10. Define summable sequence of complex numbers. Hence, define $l^1(Z)$.

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Let $\hat{u} = (\sqrt{2}, 1, 0, 1)$ and $\check{v} = (0, 1, \sqrt{2}, -1)$
(a) Find u and v
(b) Construct an orthonormal basis for $l^2(Z_4)$ using u and v
12. Suppose N is divisible by 2 and $u_1 \in l^2(Z_N)$
Define $u_2 \in l^2(Z_{N/2})$ by $u_2(n) = u_1(n) + u_1(n + \frac{N}{2})$ Then prove that $\hat{u_2}(m) = \hat{u_1}(2m)$.
13. Describe the analysis phase and synthesis phase in a filter bank diagram through an example.
14. Let M$\in Z$ and $\{x_n\}_{n=M}^{\infty}$ be a sequence in a normed space X. Then
i) Define the Convergence of $\{x_n\}_{n=M}^{\infty}$ to an element x of X.
ii) When we say $\{x_n\}_{n=M}^{\infty}$ is a Cauchy sequence?
iii) Show that if $\{x_n\}_{n=M}^{\infty}$ is a Convergent sequence,then it is a cauchy sequence.
iv) If X is an innerproduct space, when we say X is a Hilbert space?
15. Let H be a Hilbert space.

i) If $\{f_n\}_{n=1}^{\infty}$ is a sequence in H and $f \in H$, prove that $f_n \to f$ implies $< f_n, g > \to < f, g >$ for all $g \in H$.
ii) If $\{a_j\}_{j \in Z}$ is an orthonormal set in H and $z = (z(n))_{n \in Z} \in l^2(Z)$, prove that $< \sum_{j \in Z} z(j) a_j, a_m >= z(m)$ for all $m \in Z$.

16. (i) Prove that $l^1(Z)$ is not an innerproduct space.
(ii) Prove that $l^1(Z) \subset l^2(Z)$.
(iii) Give an example of $z \in l^2(Z)$ such that $z \notin l^1(Z)$.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. (a) Suppose $z, w \in l^2(Z_N)$. Then prove that for each m, $(z * w)^{\wedge}(m) = \hat{z}(m)\,\hat{w}(m)$
(b) Let z=(1,0,1,0) and w=(0,1,0,1) be two elements of $l^2(Z_4)$. Find $z * w$.
(c) Find $(z * w)^{\wedge}$
(d) Find $\hat{z}$ and $\hat{w}$ and verify $(z * w)^{\wedge}(m) = \hat{z}(m)\hat{w}(m)$ for m=0,1,2,3.

OR

2. (a) Let $w \in l^2(Z_N)$. Then prove that $\{R_k w\}_{k=0}^{N-1}$ is orthonormal basis for $l^2(Z_N)$ if and if $|\hat{w}(n)| = 1$ for all $n \in Z_N$
(b) If $B = \{R_k w\}_{k=0}^{N-1}$ is an orthonomal basis for $l^2(Z_N)$. Prove that $[z]_B = z * \tilde{w}$.

18.1. Describe real shannon wavelet system.

OR

2. Describe Daubechie's D_6 wavelet system on Z_N.

19.1. Suppose A=$\{a_j\}_{j \in Z}$ is an orthonormal set in a Hilbert space H and S=S_A. Define the transformation P_s. Prove the following
i) $P_S(f) \in S$
ii) P_S is linear
iii) $P_S(s) = s$ for any $s \in S$
iv) $< f - P_S(f), s >$ =0 for any $f \in H$ and $s \in S$
v) For any $f \in H$, $s \in S$, $||f - P_S(f)|| \leq ||f - s||$ with equality holds only if $s = P_S(f)$.

OR

2. Suppose $f : [-\pi, \pi) \to C$ is both continuous and bounded, say $|f(\theta)| \leq M$ for all θ. If $< f, e^{in\theta} >= 0$ for all $n \in Z$, prove that $f(\theta)$=0 for all $\theta \in [-\pi, \pi)$.

20.1. Suppose $u, v \in l^1(Z)$. Then prove that $B = \{R_{2k} v\}_{k \in Z} \bigcup \{R_{2k} u\}_{k \in Z}$ is a complete orthonormal set in $l^2(Z)$ if and only if $A(\theta)$ is unitary for all $\theta \in [0, \pi)$.

OR

2. Suppose $u_l, v_l \in l^1(Z)$ for each $l \in N$ and the system matrix $A_l(\theta)$ is unitary for all $\theta \in [0, \pi)$. Define $f_1 = u_1, g_1 = u_1$ and for $l \in N, l \geq 2$, define $f_l = g_{l-1} * U^{l-1}(v_l)$

and $g_l = g_{l-1} * U^{l-1}(u_l)$. If we define V_{-l} for $l \in N$ as
$V_l = [\sum_{k \in Z} z(k) R_{2^l k} g_l : z = (z(k))_{k \in Z} \in l^2(Z)]$ and if $\bigcap_{l \in N} V_{-l} = \{0\}$, prove that
$B = \{R_{2^l k} f_l : k \in Z, l \in N\}$ is a complete orthonormal set in $l^2(Z)$.

(10 x 4 = 40)