Reg. No .... Name **18P428**

M Sc DEGREE END SEMESTER EXAMINATION - MARCH 2018

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT18EL ; COMBINATORICS**

(For Regular - 2016 admission)

Time : Three Hours Max. Marks: 75

Section A

Answer all the following (1.5 marks each)

1. Derive the relation between permutations and combinatons?
2. Let $n, r\epsilon\ N$ with $r \leq n$. Prove the following dentition: $P_r^n = \frac{n}{n-r} P_r^{n-1},\ where\ r < n$
3. Find the number of positive divisors of n (inclusive of n) which are multiples of 3, where n = 630.
4. Prove that among any group of 13 people, there must be at least 2 whose birthdays are in the same month.?
5. Find that any among any group of 3000 people, there are at least 9 who have the same birthday?.
6. Define Ramsey number.
7. Let S = {1,2, ... ,100}. Find the number of integers in S which are divisible by 5.?
8. Explain generalized principle of inclusion and exclusion?
9. Let a_r be the number of ways of distributing r identical objects into n distinct boxes. Then find the generating function for (a_r)
10. Define exponential generating function for the sequence of numbers (a_r)?.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Find the number of ordered pairs (x,y) of integers such that sum of their squres is less than or equal to 5.?
12. Let X= {1,2...., 100} and let { (a,b,c) | a,b,c,$\epsilon X, a < b\ and\ a < c$ }. find |S|?
13. Let $X \subseteq 1, 2, \ldots .99$ and $|X| = 10$. Show that it is possible to select two disjoint nonempty proper subsects Y, Z of X such that $\sum(y|y\ \epsilon\ Y) = \sum(x|x\ \epsilon\ Z)$.
14. Let $n\epsilon N$,and let $n = p_1^{m_1} p_2^{m_2} \ldots p_k^{m_k}$, be its prime fractorization then show that $\varphi(n) = n \prod_{i=1}^{k} \left(1 - \frac{1}{p_i}\right)$.
15. Find the number of integer solutions to the equation $x_1 + x_2 + x_3 = 28$,where $3 \leq x_1 \leq 9, 0 \leq x_2 \leq 8, 7 \leq x_3 \leq 17$; using propertis?
16. In how many ways can 4 of the letters from PAPAYA be arranged using generating functons

(5 x 4 = 20)

Section C

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17.1. a) If there must be at least one person in each table, in how many ways can 6 people be seated?
(i) around two tables?
(ii) around three tables?
(We assume that the tables are indistinguishable.)
b) Show that s(r, n) = s(r - 1, n - 1) + (r - 1)s(r - 1, n)

OR

2. a) Find the number of ways to choose a pair {a, b} of distinct numbers from the set {1,2, ... ,50} such that
(i) | a - b | = 5;
(ii) | a - b | ≤ 5
b) There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
(i) there are no restrictions?
(ii) the 5 girls must be together (forming a block)?
(iii) no 2 girls are adjacent?
(iv) between two particular boys A and B, there are no boys but exactly 3 girls?

18.1. a) Prove that for all integers $p, q \leq 2$, $R(p,q) \leq R(p-1,q) + R(p,q-1)$.
b) Seventeen people correspond by mail with one another - each one with all the rest. In their letters only three different topics are discussed. Each pair of correspondents deals with only one of these topics. Prove that there are at least three people who write to one another about the same topic.

OR

2. a) Show that for all intergers $p, q \geq 2$, if R(p-1,q) and R(p,q-1) are even , then R(p,q) $\leq$ R(p-1,q) + R(p,q-1) - 1.
b) Six points are in general position in space (no three in a line, no four in a plane). The fifteen line segments joining them in pairs are drawn, and then painted with some segments red and the rest blue. Prove that some triangle has all its sides the same colour.

19.1. a) Find the number of nonnegative integer solutions to the equation $x_1 + x_2 + x_3 = 15$; using GPIE $x_1 \leq 5,\ x_2 \leq 6,\ x_3 \leq 7$.
b) Prove that for any $n \epsilon N, \lim_{x\to\infty} \binom{D_n}{n!} = e^{-1} \simeq 0.367$

OR

2. Explain the problem of Tower of Hanoi with diagrams

20.1. a) Solve the recurrence relation $a_n - 7a_{n-1} + 15a_{n-2} - 9a_{n-3} = 0$; Given that $a_0 = 1$, $a_1 = 2$ and $a_2 = 3$.
b)Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$; Given that $a_0 = 1$ and $a_1 = 0$.

OR

2. a) Solve the recurrence relation $a_n - 3a_{n-1} = 2 - 2n^2)$; Given that $a_0 = 3$.
b) Prove that for each $n \epsilon N$, the number of partitions of n into parts each of which appears at most twice, is equal to the number of partitions of n into parts the sizes of which are not divisible by 3.

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