Reg. No .... Name **18P416**

M Sc DEGREE END SEMESTER EXAMINATION - MARCH 2018

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT17EL ; MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS**

(For Regular 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A
Answer all the following (1.5 marks each)

1. State Fourier Integral theorem
2. Define Integral transforms
3. Statethe exponential form of Fourier series
4. Define directional derivative
5. If $f'(c;u)$ exists in every direction u, then in particular all the partial derivatives $D_1f(c)....D_kf(c)$ exist. what about the converse? is it not true?.
6. If $f=u+iv$ is a complex-valued function with a derivative at a point z in C, then $J_f(z)=If'(z)|^2$.
7. Define Jacobian determinant
8. Find ths stationary points of the function x^3+y^3+3xy
9. Define the term support
10. Define product of two basic k-forms

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Express the function f(x)={ $\begin{matrix} 1 & for|x|\le 1 \\ 0 & for|x|\ge 1 \end{matrix}$ as a fourier integral. Hence evaluate $\int_0^\infty \frac{sin\lambda cos\lambda x}{\lambda}d\lambda$
12. Obtain the Fourier series of the function $f(x)=1/4(\pi-x)^2$, $0\le x\le 2\pi$
13. Find the directional derivative of the function $f=x^2-y^2+2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ where Q is the point $5,0,4$
14. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube
15. Find the points on the surface $z^2=xy+1$ nearest to the origin
16. Find $J_f(r,\theta,\phi)$ where $f(r,\theta,\phi)$ is defined by $x=rsin\theta cos\phi, y=rsin\theta sin\phi, z=rcos\theta$

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. State and prove Weisrstrass Approximation theorem

OR

2. Find the Fourier transform of $f(x)= \begin{matrix} 1-x^2 & when|x|<1 \\ 0 & when|x|>1 \end{matrix}$ Use it to evaluate

$\int_0^\infty \frac{xcosx - sinx}{x^3} cos\frac{x}{2} dx$

18.1. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative$f'(b)$. Then prove that the composite function$h = fog$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b)og'(a)$, the composition of the linear functions $f'(b)$ and $g'(a)$.

OR

2. a) Show that $m(SoT) = m(S)m(T)m(SoT) = m(S)m(T)$.
b) Compute the gradient vector $\nabla f(x, y, z)$ at the point $1, 3, 5)$ of the function
$f(x, y, z) = \frac{x^3 y^5 z^5}{1 + x^4 + y^3 + z^2}$

19.1. If both partial derivatives $D_r f$ and $D_k f$ exist in an n-ball $B(c;\delta)$ and if both are differentiable at c, thenshow that $D_r, kf(c) = D_k, rf(c)$.

OR

2. Let $B = B(a;r)$ be an n-ball in R^n, let δBdenote its boundary,$\delta B = x : ||x - a|| = r$, and let $B = BU\delta B$ denote its closure. Let $f = (f_1, \ldots, f_n)$ be continuous on B, and assume that all the partial derivatives $Djf_i(x)$ exist if $x \in B$. Assume further that $f(x) \neq f(a)$ if $x \in \partial B$ and that the Jacobian determinant $J_f(x) \neq 0$ for each x in B. Then prove that $f(B)$, the image of B under f, contains an n-ball with center at$f(a)$

20.1. a)Suppose T is a $1-1$ $\zeta^{I}-$ mapping of an open set $E \subset R^k$ into R^k such that $J_T(x) \neq 0$ $\forall x \in E$. If f is a continuous function onR^k whose support is compactand lies in $T(E)$, then prove that $\int_{R^k} f(y)dy = \int_{R^k} f(T(x))|J_T(x)|dx$
b) Evaluate$\int\limits_0^1 \int\limits_0^{\sqrt{1-x^2}} \int\limits_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1 - x^2 - y^2 - z^2}}$, by changing to spherical polar co-ordinates

OR

2. a)Suppose $w = \sum\limits_I b_I(x)dx_I$ is the standared representationof a $k - form\ w$ in an open set $E \in R^n$. If$w = 0$in E, then prove that $b_I(x) = 0$ for every increasing $k-$ index I and for every $x \in E$
b)Suppose T is a $1-1$ $\zeta^{I}-$ mapping of an open set $E \subset R^k$ into R^k such that $J_T(x) \neq 0$ $\forall x \in E$. If f is a continuous function onR^k whose support is compactand lies in $T(E)$, then prove that $\int_{R^k} f(y)dy = \int_{R^k} f(T(x))|J_T(x)|dx$

(10 x 4 = 40)