Reg. No .... Name **18P255**

M Sc DEGREE END SEMESTER EXAMINATION - APRIL 2018

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT10 ; REAL ANALYSIS**

(Common for Regular - 2017 Admission & Supplementary - 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A
Answer All (1.5 marks each)

1. Let f be a function defined on $[a,b]$. Suppose that $|f(x)-f(y)| \leq M|x-y|,\ \forall x,y \in [a,b]$. Prove that f is of bounded variation on $[a,b]$.
2. Show that a polynomial is always a function of bounded variation on every compact interval.
3. Prove that if $f \in \mathscr{R}$, then $f^2 \in \mathscr{R}$. Is converse true? Justify.
4. Prove that $\underline{\int_a^b} f d\alpha \leq \overline{\int_a^b} f d\alpha$.
5. If f is continuous on [a,b], then prove that $f \in \mathscr{R}(\alpha)$.
6. Show by an example that there is a sequence of continuous functions, whose limit is discontinuous.
7. Discuss the uniform convergence of the sequence of functions $\{f_n(x)\}$, where $f_n(x) = \frac{x}{n}$, $x \in \mathbb{R}$.
8. State Weierstrass M-test.
9. Discuss the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2+1}$
10. Prove that $\lim_{x \to \infty} x^n e^{-x} = 0$.

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Prove that the number of discontinuities of a monotone function on [a,b] is countable.
12. Let f be a function of bounded variation on [a,b]. If $c \in [a,b]$, prove that f is a function of bounded variation on [a,c] and [c,b].
13. Describe the method for calculating the total variation of a differentiable function on [a,b]
14. If $f \in \mathscr{R}(\alpha)$ on [a,b], $m \leq f(x) \leq M, \varphi$ is continuous on [m,M], and $h(x) = \varphi(f(x))$ on [a,b]. Prove that $h \in \mathscr{R}(\alpha)$ on [a,b].
15. For $n = 1,2,3,\ldots; x$ is real, put $f_n(x) = \frac{x}{1+nx^2}$. Show that $\{f_n\}$ converges uniformly to a function f, and that the equation $f^{'}(x) = \lim_{n \to \infty} f_n^{'}(x)$ is correct if $x \neq 0$, but false if $x = 0$
16. Show by an example that there exist a function f whose derivative at $x = 0$ exists for all orders but $f(x) \neq f(0) + \frac{f^{'}(0)}{\ldots} x + \frac{f^{''}(0)}{\ldots} x^2 + \ldots$

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17. Let f and g be complex-valued functions defined as follows:

$$f(t) = e^{2\pi it} \quad \text{if} \quad t \in [0,1]; \quad g(t) = e^{2\pi it} \quad \text{if} \quad t \in [0,2]$$

a. Prove that f and g have the same graph but not equivalent.
b. Prove that the length of g is twice that of f.

OR

18. Prove that the graph of $f(x) = \begin{cases} x\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is not rectifiable on [0,1].

19. Let $f(x) = \begin{cases} 1 & \text{if } x \in C \\ 0 & \text{if } x \notin C \end{cases}$, for all $x \in$ [0,1], where C is the Cantor set. Prove that $f \in \mathscr{R}$ on [0,1].

OR

20. If $f \in \mathscr{R}(\alpha)$ on [a,b], $m \leq f(x) \leq M, \varphi$ is continuous on [m,M], and $h(x) = \varphi(f(x))$ on [a,b]. Prove that $h \in \mathscr{R}(\alpha)$ on [a,b]. Prove also that if $f : [a,b] \longrightarrow [0,\infty)$ is Riemann integrable, then $\sqrt{f}$ is also Riemann integrable.

21. Prove that the series $\sum \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval, but does not converge absolutely for any value of x.

OR

22. State the Stone-Weierstrass theorem. Prove that if f is continuous on [0,1] and $\int_a^b f(x)x^n dx = 0$ for $(n = 0,1,2\ldots)$, then $f(x) = 0$ on [0,1].

23. Suppose the series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ converge in the segment $S = (-R,R)$. Let E be the set of all $x \in S$ at which $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$. If E has a limit point in S, then prove that $a_n = b_n$ for $n = 0,1,2,\ldots$.

OR

24.
a. If $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$, prove that $E(x) = e^x \ \forall x \in \mathbb{R}$
b. If z is a complex number wiht $|z| = 1$, prove that there is a unique $t \in [0,2\pi)$ such that $E(it) = z$.

(10 x 4 = 40)

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