Reg. No .... Name **18P231**

MSc DEGREE END SEMESTER EXAMINATION- APRIL 2018

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT08 ; ADVANCED COMPLEX ANALYSIS**

(For Regular - 2017 Admission & Supplementary - 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A
Answer any 10 (1.5 marks each)

1. Define a Reimann's zeta function.
2. Prove that $\lceil \frac{1}{2} = \lceil \pi$
3. Expand $(1-z)^{-m}$, (m is a positive integer) in powers of z
4. State Hadamard's theorem
5. State Jenson's Formula.
6. Define a Normal family.
7. State Riemann mapping theorem.
8. Define a free boundary arc.
9. Define a periodic function with example.
10. Show that an elliptic function without poles is a constant.

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Prove that the necessary and sufficient condition that the infinite product $\pi_{n=1}^{\infty}(1+a_n), a_n \neq -1$ is convergent is the convergence of the series $\sum_{n=1}^{\infty} \log(1+a_n)$, where each log has its principle value.
12. Prove that the infinite product $\pi_{n=1}^{\infty}(1+a_n)$ with $1+a_n \neq 0$ converge simultaneously with the series $\sum_{n=1}^{\infty} \log(1+a_n)$ whose terms represents the value of the principal branch of the logarithm.
13. Prove that zeta function can be extended to a meromorphic function in the whole complex plane whose only pole is a single pole at $S=1$ with residue 1.
14. Show that a family $\mathcal{F}$ is normal if and if its closure $\bar{\mathcal{F}}$ w.r.t the distance function $\rho(f,g) = \sum_{k=1}^{\infty} \frac{\delta_k(f,g)}{2^k}$ is compact.
15. Suppose that the boundary of a simply connected region Ω contains a line segment γ as one sides free boundary arc. Prove that the function $f(z)$ which maps Ω onto the unit disc can be extended to a function which is analytic and one-one on $\Omega \cup \gamma$. Also image of γ is an arc γ' on the unit circle.
16. Prove that $\mathcal{P}(2z) = \frac{1}{4}\left[\frac{\mathcal{P}"(z)}{\mathcal{P}'(z)}\right]^2 - 2\mathcal{P}(z)$.

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17. State and prove Abel's Theorem

OR

18. State and prove the Weierstears Theorem for canonical product

19. Derive Poisson-Jensen formula.

OR

20. Derive Legendre's duplication formula.

21. a. Prove that a continuous function $u(z)$ which satisfies the condition $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + re^{i\theta})d\theta$ is necessarily harmonic .
b. State Harnack's Principle by proving the corresponding Harnack's inequality.

OR

22. Suppose that the boundary of a simply connected region Ω contains a line segment of one sides for boundary arc. Then prove that the function $f(z)$ which maps Ω onto the unit disc can be extended to a function which is analytic and one one on $\Omega \cup f$. The image of γ is an arc γ on the unit circle.

23. Show that any even elliptic function with periods w_1, w_2 can be expressed in the form $C\prod_{k=1}^{n} \frac{\mathcal{P}(z)-\mathcal{P}(a_k)}{\mathcal{P}(z)-\mathcal{P}(b_k)}$, provided 0 is neither a zero nor a pole. What is the corresponding form if the function either vanishes or becomes infinite at origin.

OR

24. a. Show that an elliptic function without poles is a constant.
b. Prove that the sum of the residues of an elliptic function is zero

(10 x 4 = 40)