Reg. No .... Name **18P218**

MSc DEGREE END SEMESTER EXAMINATION- APRIL 2018

SEMESTER 2 : MATHEMATICS

COURSE : **16P2MATT07 ; ADVANCED TOPOLOGY**

(For Regular - 2017 Admission & Supplementary - 2016 Admission)

Time : Three Hours Max. Marks: 75

Section A
Answer any 10 (1.5 marks each)

1. State Urysohn's Lemma.
2. State the Urysohn characterization of normality.
3. If a space X has the property that for any two mutually disjoint closed subsets A, B of it there exists a continuous function $f : X \to [0,1]$ such that $f(x) = 0\ \forall x \in A$ and $f(x) = 1\ \forall x \in B$, then prove that X is normal.
4. Give an example of a metric space which is not 2nd countable.
5. Define a subnet of a net S in X.
6. If every net in a space X can converge to any point in it, then prove that X is indiscrete.
7. If a space X is Hausdorff, prove that no filter on X can converge to more than one point in it.
8. Define one point compactification of the space (X, T).
9. Prove that one point compactification (X^+, T^+) of the space (X, T) is compact
10. Let $X^+ = XU\{\infty\}$ be the one point compactification of the space X. Prove that if $\{\infty\}$ is open in X^+, then X is compact.

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Suppose a topological space X has the property that for every closed subset A of X, every continuous real valued function on A has a continuous extension to X. Then prove that X is normal.
12. Let A be a closed subset of a normal space X and suppose $f : A \to (-1,1)$ is continuous. Then prove that there exists a continuous function $F : X \to (-1,1)$ such that $F(x) = f(x),\ \forall\ x \in A$.
13. Let $\{f_i : X \to Y_i | i \in I\}$ be a family of functions which distinguishes points from closed sets in X. Prove that the corresponding evaluation function $e : X \to \pi Y_i$ is open when regarded as function from X onto $e(X)$.
14. Show that a space is compact iff every universal net in it is convergent.
15. Prove that a space is Hausdorff iff every ultra filter converges to atmost one point in it.
16. Prove that a space X is compact iff every filter on X has a cluster point in X.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17. Prove that a product of spaces is connected iff each co-ordinate space is connected.

18. Prove that a topological product is T_0, T_1, T_2 or regular iff each co-ordinate space has the corresponding property

19. If the product space is non-empty, prove that each co-ordinate space is embeddable in it and hence prove that if a topological product is T_0, T_1, T_2 or regular, then each co-ordinate space has the corresponding property.

OR

20. Define the evaluation function \`e' of the indexed family of functions. $\{f_i : X \to Y_i; i \in I\}$ and prove that it is the only function from X into πY_i such that $\pi_i \circ e = f_i; \forall i \in I$. Also prove that e is 1-1 iff the family $\{f_i\}$ distinguishes points and e is continuous iff each f_i is continuous.

21. State and prove Urysohn Embedding Theorem.

OR

22. Let A be a subset of a space X and let $x \in X$. Prove that $x \in \bar{A}$ iff there exists a net in A which converges to x. Also prove that A is closed iff limits of all nets in A are in A and A is open iff no net in the complement, $X - A$ can converge to a point in A.

23. Prove that every countably compact metric space is second countable. Also prove that a metric space is compact iff it is countably compact.

OR

24. Prove that countable compactness is preserved under continuous function and a countably compact metric space is compact. Also prove that every continuous, real-valued functions on a countably compact space is bounded and attains its extrema.

(10 x 4 = 40)

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