Reg. No .... Name **18P157**

MSc DEGREE END SEMESTER EXAMINATION - NOVEMBER 2018

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT05 : COMPLEX ANALYSIS**

(For Regular - 2018 Admission & Supplementary - 2017 & 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer any 10 (1.5 marks each)

1. If $T_1(z) = \dfrac{z+2}{z+3}$ and $T_2(z) = \dfrac{z}{z+1}$. Find $T_1T_2(z)$
2. Prove that the map $w = \bar{z}$ is not conformal
3. Evaluate the cross ratio $(2, \infty, 1-i, 3+i)$
4. Define winding number
5. Evaluate $\int_{|z|=2} \dfrac{e^z}{z-3} dz$
6. Evaluate $\int_C \dfrac{dz}{z-a}$ where C is any simpleclosed curve and $z = a$ is i)inside C ii) outside C
7. True or faulse "A rational function has no singularities other than poles" Justify?
8. Find the algebraic order of $\dfrac{z^4+z+1}{(z-4)^5}$ at its poles.
9. Find the number of poles of $f(z) = \dfrac{1}{z(z^2+3)(z^2+2)^3}$ inside the circle $|z| = 1$.
10. Write the Poisson integral of U

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Show that the resultant (or product) of two bilinear transformations is a bilinear transformation
12. Give a precise definition of a single valued branch of $\sqrt{z}$ and prove that it is analytic
13. State and prove Cauchy's Integral formula
14. Show that the order of a zero of a polynomial equals the order of its first non-vanishing derivative.
15. State Poisson's formula and prove that $\int_0^{2\pi} \dfrac{e^{cos\phi} cos(sin\phi)}{5-4cos(\theta-\phi)} d\phi = \dfrac{2\pi}{3} e^{cos\theta} cos(sin\theta)$
16. Evaluate $\int_C \dfrac{sin\pi z^2 + cos\pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. Describe the Riemann surface associated with $w = \frac{1}{2}(z + \frac{1}{z})$

OR

2. Find the linear transformation which carries the points $z = -2, 0, 2$ into the points $w = 0, i, -1$ respectively. Also find the transformation which carries $z = 0, -1, \infty$ into the points $w = -1, -2 - i, i$

18.1. State and prove Cauchy's theorem for a rectangle.

OR

2. a. State and prove the lemma for higher derivatives
b. State and prove Liouvillie's theorem

19.1. State and prove Schwarz lemma

OR

2. Suppose that $f(z)$ is analytic at z_0, $f(z_0) = w_0$ and that $f(z) - w_0$ has a zero of order n at z_0. Then prove that if $\xi > 0$ is sufficiently small, $\exists$ a corresponding $\delta > 0$ such that for all a with $|a - w_0| < \delta$ the equation $f(z) = a$ has exactly n roots in the disk $|z - z_0| < \xi$.

20.1. State and prove Poisson's Integral formula

OR

2. Evaluate $\int_0^\infty \frac{cosx}{x^2 + a^2} dx$, a real

(10 x 4 = 40)