Reg. No .... Name **18P144**

MSc DEGREE END SEMESTER EXAMINATION - NOVEMBER 2018

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT04 : ORDINARY DIFFERENTIAL EQUATIONS**

(For Regular - 2018 Admission & Supplementary - 2017 & 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A
Answer any 10 (1.5 marks each)

1. Show that the vector functions $\varphi(t) = \begin{bmatrix} e^t \\ 2e^t \end{bmatrix}, \psi(t) = \begin{bmatrix} e^{3t} \\ 4e^{3t} \end{bmatrix}$ are linearly independent on any interval $a \leq t \leq b$.
2. Is $(e^{3t}, -e^{3t})$ a solution of $\frac{dx}{dt} = 2x - y$. Justify your answer.
$$\frac{dy}{dt} = 3x + 6y$$
3. Does there exist any homogeneous linear system of two unknown functions on an interval $-\pi/2 \leq t \leq \pi/2$ such that its wronskian of two solutions is W(t) = sin(t) on $-\pi/2 \leq t \leq \pi/2$. Justify your answer.
4. Define Cauchy product of two series and state a necessary condition for the cauchy product of two convergent series to be convergent.
5. Determine the nature of the singularity of the point $x = 0$ of the differential equation $x^4y'' + (sinx)y = 0$.
6. Show that $sin(x)$ and $sin(2x)$ are orthogonal with respect to the weight function $r(x) = 1$ on the interval $0 \leq x \leq \pi$.
7. The sequence of functions $\{sin(nx)\}_{n=1}^{\infty}$ is orthonormalized with respect to the weight function $r(x) = 1$ on the interval $0 \leq x \leq \pi$. State true or false and justify your answer.
8. Find $L[4sin(2x) + 6x]$.
9. Find $L[4sin(x)cos(x) + 2e^{-x}]$.
10. Calculate $\int_0^{\infty} \frac{e^{-ax}sin(bx)}{x}dx$.

(1.5 x 10 = 15)

Section B
Answer any 4 (5 marks each)

11. Show that any linear combination of two solutions of the homogeneous linear system $\frac{dx}{dt} = a_1x + a_2y, \frac{dy}{dt} = b_1x + b_2y$ is itself a solution of that system.
12. Find the general solution of the system $\frac{dx}{dt} = x + 2y, \frac{dy}{dt} = 3x + 2y$.
13. Show that $x = 0, 1$ are regular singular points of Gauss's Hypergeometric equation.
14. Find characteristic values and characteristic functions of the Strum - Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\frac{\pi}{2}) = 0$

15. Find $L[sin^2(ax)]$ and $L[cos^2(ax)]$ without integrating. How are these two transforms related to one another?

16. Show that $L\left[\frac{f(x)}{x}\right] = \int_p^\infty F(s)ds$ and hence calculate $\int_0^\infty \frac{e^{-ax}sin(bx)}{x}dx$.

(5 x 4 = 20)

Section C
Answer any 4 (10 marks each)

17.1. Find particular solution of the linear system $\frac{dx}{dt} = 3x + 5y,\ \frac{dy}{dt} = -2x + 5y$ that satisfies the initial conditions x(0) = 5 and y(0) = -1.

OR

2. Find the general solution of the homogeneous linear system $\frac{dx}{dt} = \begin{bmatrix} 7 & 4 \\ -1 & 3 \end{bmatrix} x.$

18.1. Verify that 0 is an ordinary point and then find the power series solution of the differential equation $y'' - y' + xy = 0$.

OR

2. The equation $x^2y'' - 3xy' + (4x + 4)y = 0$ has only one Frobenius series solution. Find it.

19.1. Consider the Strum-Liouville problem $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + [q(x) + \lambda r(x)]\, y = 0$ with boundary conditions $A_1y(a) + A_2y'(a) = 0$ and $B_1y(b) + B_2y'(b) = 0$ where A_1, A_2, B_1, B_2 are real constants such that A_1 and A_2 are not both zero and B_1 and B_2 are not both zero. Show that the characteristic funcions corresponding to distinct characteristic values are orthogonal with respect to the weight function $r(x)$ on the interval $a \leq x \leq b$.

OR

2. Find characteristic values and characteristic functions of the Strum - Liouville problem $\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{\lambda}{x}y = 0, y'(1) = 0, y'(e^{2\pi}) = 0$ where the parameter λ is nonnegative.

20.1. Calculate the Laplace transfom of $\sqrt{x}$ and hence find $L[x^{3/2}]$.

OR

2. Define a cycloid and show that the curve of descent is a cycloid when $T(y) = T_0$, a constant.

(10 x 4 = 40)