Reg. No .... Name **18P117**

M.Sc DEGREE END SEMESTER EXAMINATION - NOVEMBER 2018

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT02 : BASIC TOPOLOGY**

(For Regular - 2018 Admission & Supplementary - 2016 / 2017 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer any 10 (1.5 marks each)

1. Is arbitrary intersection of open sets open? Justify.
2. Let X be a set and $\{\mathscr{T}_i, i \in I\}$ be an indexed collection of topologies on X. Show that $\mathscr{T} = \cap_{i \in I} \mathscr{T}_i$ is a toplogy on X.
3. Define closure. What is the closure of set of rational numbers $\mathscr{Q}$ under the usual topology on $\mathscr{R}$?
4. Show that composition of continuous functions is also continuous.
5. Show that the inclusion function is continuous.
6. Define a compact set A in a space $(X, \mathscr{T})$. Give an example of a set that is not compact.
7. Show that every path connected space is connected.
8. Define a component. Show that components are closed sets.
9. Define (i) T_1 space (ii) Normal Space.
10. Define (i) T_2 space (ii) Completely regular Space.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. If a space is second countable then prove that every open cover of it has a countable subcover.
12. Prove that every second countable space is separable.
13. Show that closed subspace of compact space is compact.
14. A space is locally conneceted at a point x if and only if for every neighbourhood N of x, the component of N containing x is a neighbourhood of x.
15. Show that closure of a connected space is connected. Is the converse true. Justify your answer.
16. Show that every Tychonoff space is regular.

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. Prove that Metrisibily is a hereditary property.

OR

2. Establish three equivalent condition for a topological space $\mathscr{T}$ to have a base $\mathscr{B}$.

18.1. Let $[(X_i, \mathscr{T}_i), i = 1, 2, \ldots n]$ be a collection of topological spaces and $(X, \mathscr{T})$ their topological product. Prove that each projection π_i is continuous. Also show that if Z is any space then the fuction $f : Z \to X$ is continuous if and only if $\pi_i o f : Z \to X_i$ is continuous for all $i = 1, 2, \ldots n$.

OR

2. Prove that product topology on $R \times R$ coincides with the metric topology on $R \times R$.

19.1. (a) Let $X_1 \ldots X_n$ be topological spaces and X their topological product. Suppose X_i is locally connected at a point x_i for $i = 1 \ldots n$. Let $x = (x_1 \ldots x_n) \in X$. Prove that X is locally connected at x.

(b) Prove that an open subspace of a locally connected space is locally connected.

OR

2. For a topological space X show that the following are equivalent:

   (a) X is locally connected.

   (b) Components of open subsets of X are open in X.

   (c) X has a base consisting of conneceted subsets.

   (d) For every $x \in X$ and every neighbourhood N of x there exists a connected open neighbourhood M of x such that $M \subset N$.

20.1. (a)Show that the axioms T_0, T_1, T_2, T_3 and T_4 form a hierarchy of progressively stronger condition.

(b) Every continuous, one to one function from a compact space onto a Hausdorff space is an embedding.

OR

2. Let X be a completely regular space. Suppose F is a compact subset of X, C is a closed subset of X and $F \cap C \neq \emptyset$. Show that there exist a continuous function from X to $[0, 1]$ which takes the value 0 at all points of F and the value 1 at all points of C.

(10 x 4 = 40)