Reg. No .... Name **18P103**

MSc DEGREE END SEMESTER EXAMINATION - NOVEMBER 2018

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT01 : LINEAR ALGEBRA**

(For Regular - 2018 Admission & Supplementary - 2017 & 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer any 10 (1.5 marks each)

1. Let V be a vector space over the field F. Show that the intersection of any collection of subspaces of V is a subspace of V.
2. Verify whether $(3,-1,0,-1)$ is in the subspace of $\mathbb{R}^4$ spanned by the vectors $(2,-1,3,2),(-1,1,1,-3)$ and $(1,1,9,-5)$.
3. Is the set of vectors $\alpha = (a_1,\ldots,a_n) \in \mathbb{R}^n$ such that $a_1a_2 = 0$, a subspace of $\mathbb{R}^n$?
4. Define annihilator of a subset S of a vector space V. What is the annihilator of $S = \{0\}$?
5. Define hyperspace in a vector space. Give an example.
6. Is there a linear transformation T from R^3 into R^2 such that $T(1,-1,1) = (1,0)$ and $T(1,1,1) = (0,1)$? Justify.
7. Prove that the determinant of a triangular matrix is the product of its diagonal entries.
8. Define the terms characteristic value, characteristic vector and characteristic space with respect to a linear operator T on a vector space V.
9. Let E be a projection on V with range R and null space N. Show that $V = R \oplus N$.
10. If $T^2 = T$ show that T is diagonalizable.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Let V be the set of all pairs (x,y) of real numbers, and let F be the field of real numbers. Define $(x,y)+(x_1,y_1) = (x+x_1, y+y_1)$ and $c(x,y) = (cx,y)$.
    Is V with these operations, a vector space over the field of real numbers?
12. If W is a subspace of a finite-dimensional vector space V, show that every linearly independent subset of W is finite and is part of a basis for W. Hence show that if W is a proper subspace of a finite-dimensional vector space V, W is finite -dimensional and $dim\, W < dim\, V$.
13. Show that $\{(1,2),(3,4)\}$ is a basis for $\mathbb{R}^2$. Let T be the unique linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ such that $T(1,2) = (3,2,1)$ and $T(3,4) = (6,5,4)$. Find $T(1,0)$
14. Let F be a subfield of the complex numbers and let T be the function from F^3 into F^3 defined by $T(x_1,x_2,x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$.

(a) Verify that T is a linear transformation.
(b) If (a, b, c) is a vector in F^3, what are the conditions on a, b and c that the vector be in the range of T? What is the rank of T?

15. Let W be an invariant subspace under a linear operator T on a finite dimensional vector space V and let α be any element of V. Show that the T-conductor of α into W divides the minimal polynomial for T.

16. Find the characteristic values and characteristic vectors of the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

(5 x 4 = 20)

Section C

Answer any 4 (10 marks each)

17.1. Let m and n be positive integers and let F be a field. Suppose W is a subspace of F^n and $\dim W \leq m$. Show that there is precisely one $m \times n$ row-reduced echelon matrix over F which has W as its row space.

OR

2. Let V be the vector space of all 2×2 matrices over the field F.Let W_1 be the set of matrices of the form $\begin{bmatrix} x & -x \\ y & z \end{bmatrix}$ and let W_2 be the set of matrices of the form $\begin{bmatrix} a & b \\ -a & c \end{bmatrix}$.Prove that W_1 and W_2 are subspaces of V. Also find the dimensions of W_1,W_2,$W_1 + W_2$ and $W_1 \cap W_2$.

18.1. (a) Does there exist a linear transformation $T : R^3 \rightarrow R^2$ such that $T(1, -1, 1) = (1, 0)$ and $T(1, 1, 1) = (0, 1)$?. Justify.
(b) Let V and W be finite-dimensional vector spaces over the field F. Prove that V and W are isomorphic if and only if $\dim V = \dim W$.
(c) Let T be the linear operator on R^2 defined by $T(x_1, x_2) = (x_1, 0)$. Compute the matrix of T relative to the ordered basis $\{(1, 1), (2, 1)\}$.

OR

2. In $\mathbb{R}^3$, let $\alpha_1 = (1, 0, 1), \alpha_2 = (0, 1, -2), \alpha_3 = (-1, -1, 0)$.
(a) If f is a linear functional on $\mathbb{R}^3$ such that $f(\alpha_1) = 1, f(\alpha_2) = -1, f(\alpha_3) = 3$ and if $\alpha = (a, b, c)$, find $f(\alpha)$.
(b) Describe explicitly a linear functional f on $\mathbb{R}^3$ such that $f(\alpha_1) = f(\alpha_2) = 0$ but $f(\alpha_3) \neq 0$.
(c) Let f be any linear functional such that $f(\alpha_1) = f(\alpha_2) = 0$ and $f(\alpha_3) \neq 0$. Show that $f(2, 3, -1) \neq 0$.

19.1. Let A be an $n \times n$ matrix over the field F. Show that A is invertible over F if and only if det $A \neq 0$.When A is invertible, show that $A^{-1} = [det(A)]^{-1}$. Adj A, where Adj A is the adjoint of A.

OR

2. (a) Find the determinant of A^{10} where $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & -25 \\ 0 & 0 & 1 \end{bmatrix}$. Justify your answer.

(b) Show that a linear combination of n-linear functions is n-linear.

20.1. (a) Let T be a diagonalizable linear operator on a space V. If $c_1, \ldots, c_k$ are the distinct characteristic values of T, prove that the minimal polynomial for T is $(x - c_1)(x - c_2) \ldots (x - c_k)$.

(b) Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V.Show that T is triangulable if and only if the minimal polynomial of T is a product of linear polynomials over F.

OR

2. (a) Let T be a linear operator on a finite dimensional space V. Let $c_1, c_2, \cdots, c_k$ be the distinct characteristic values and $W_1, W_2, \cdots, W_k$ be the corresponding characteristic spaces. Prove that $\dim(W_1 + W_2 + \cdots + W_k) = \dim W_1 + \dim W_2 + \cdots + \dim W_k$.

(b) If W_1 and W_2 are subspaces of V then prove that they are independent if and only if $W_1 \cap W_2 = 0$.

(10 x 4 = 40)