Reg. No…………………………………. Name……………………………………………….

B. Sc. DEGREE END SEMESTER EXAMINATION OCTOBER/NOVEMBER 2017

SEMESTER **–1: MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS & CHEMISTRY)**

COURSE: **15U1CPMAT1: DIFFERENTIAL CALCULUS AND TRIGONOMETRY**

(Common for Regular 2017 admission and Supplementary/Improvement 2016, '15 & '14 admissions)

Time: Three Hours Max. Marks: 75

PART A

*Answer **all** questions. Each question carries **1** mark.*

1. Find the points at which the curve $y = \frac{x}{x-1}$ has slope = -1.
2. What are the points on [1, 2] at which the greatest integer function $f(x) = [x]$ is discontinuous?
3. State mean value theorem.
4. Find the absolute maximum of $f(t) = 2 - |t|$ on -1≤ t ≤ 3.
5. What are the critical points of the function $f(x) = x + \frac{1}{x}$ on $[\frac{1}{2}, 2]$.
6. Find $\frac{\partial f}{\partial x}$ if $f(x, y) = ysin(xy)$.
7. Verify w_{xy} = w_{yx} if $w = \ln(2x + 3y)$.
8. Evaluate $\frac{dw}{dt}$ at $t = \pi$ if $w = x^2 + y^2$ where $x = cost$ and $y = sint$.
9. Find the real and imaginary parts of *sin*(*x* +i*y*).
10. Show that $\sinh 2x = 2 \sinh x cosh\, x$. (1 x 10 = 10)

PART B

*Answer **any eight** questions. Each question carries **2** marks.*

11. Evaluate $\lim_{x\to 1} \frac{x^2+x-2}{x^2-x}$ if it exists.
12. Find $\frac{d^2y}{dx^2}$ if $y = \sec x$.
13. Find the absolute maximum of $f(x) = -x^2$ on [-2, 1].
14. If the displacement of a moving particle at time *t* is given by $f(t) = a + bt + ct^2$ find it's acceleration at *t* = 1.
15. Find the function *f*(*x*) whose derivative is $\sin x$ and whose graph passes through the point (0, 2).
16. Find all second order partial derivates of $f(x, y) = xcosy + ye^x$.
17. Evaluate $\frac{dy}{dx}$ at (1, 1) if $x^3 - 2y^2 + xy = 0$.
18. If $w = \frac{x}{z} + \frac{y}{z}$ where $x = \cos^2 t$, $y = \sin^2 t$ and $z = \frac{1}{t}$ evaluate $\frac{dw}{dt}$.
19. Express sinh (*x* + i*y*) in the form α +i β.
20. Show that $\sinh^{-1} z = \log(z + \sqrt{z^2 + 1})$. (2 x 8 = 16)

PART C

*Answer **any five** questions. Each question carries **5** marks*

21. A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ after t sec. Find the speed of the rock when it is 256 ft above the ground on the way up.
22. Show that the function $f(x) = |x|$ has an absolute minimum at $x = 0$. Is f differentiable at $x = 0$? Is it consistent with the first derivative theorem for local extreme values? Justify.
23. If $f'(x) = 0$ at each point of an interval I, show that $f(x) = C$ for all x in I for some constant C.
24. What is Laplace differential equation? Show that the function $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z$ satisfies Laplace equation.
25. Show that the function $w = \cos(2x + 2ct)$ is a solution of the one – dimensional wave equation $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$.
26. If $2\cos\Theta = x + \frac{1}{x}$ prove that $2\cos r\Theta = x^r + \frac{1}{x^r}$.
27. Find the general value of Θ which satisfies the equation $(\cos\Theta + i\sin\Theta)(\cos 2\Theta + i\sin 2\Theta) \ldots (\cos n\Theta + i\sin n\Theta) = 1$. (5 x 5 = 25)

PART D

*Answer **any two** questions. Each question carries **12** marks*

28. State and prove the product rule for differentiation.
29. Consider the function $f(x)$ with derivative $f'(x) = (x-1)^2(x+2)$.
    (1) What are the critical points of f?
    (2) On what intervals is f increasing or decreasing?
    (3) At what points, if any, does f assume?
    local maximum and minimum values?
30. (1) Let $w = f(x, y, z)$ be a function of three independent variables. Write the formal definition of the partial derivative $\frac{\partial f}{\partial z}$ at (x_0, y_0, z_0). Use this definition to find $\frac{\partial f}{\partial z}$ at (1,2,3) for $f(x, y, z) = x^2yz^2$.
    (2) Find $\frac{\partial w}{\partial r}$ when $r = 1$, $s = -1$ if $w = (x + y + z)^2$, $x = r - s$, $y = \cos(r + s)$ and $z = \sin(r + s)$.
31. Sum the series $\sin\alpha + x\sin(\alpha + \beta) + \frac{x^2}{2!}\sin(\alpha + 2\beta) + \ldots \infty$ (12 x 2 = 24)

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