Reg. No………………………………………….   Name……………………………………………   **16U224**

B.SC. DEGREE END SEMESTER EXAMINATION MARCH 2017

SEMESTER **– 2: MATHEMATICS (CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATIONS)**

COURSE: **15U2CRMAT2, 15U2CRCMT2: ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES**

(*Common for Regular - 2016 admission and Supplementary 2015 & 2014 admissions)*

Time: Three Hours   Max. Marks: 75

PART A

Answer **all** questions. Each question carries **1** mark.

1. What do you mean by the polar of a point with respect a circle?
2. Represent a point on the circle with centre at *a* and radius *r* using a single parameter.
3. What is latus rectum?
4. Define conjugate lines.
5. Show that $y = \pm\frac{b}{a}x$are the asymptotes of $\frac{x^2}{a^2}$ - $\frac{y^2}{b^2}$ = 1.
6. What is the real part of sin (α + βi)?
7. What is the centre of the circle $r = acos\Theta$?
8. Prove that $\cosh\Theta = \cos\,(i\,\Theta)$
9. If A is a $m \times n$ matrix, what is the maximum possible value for the rank of A?
10. Let A be singular matrix of order *n* with 1,2…*n*-1 as eigen values. What is its n^{th} eigen value?

(1 x 10 = 10)

PART B

Answer **any eight** questions. Each question carries 2 marks.

11. Find the pole of the line $lx + my + n = 0$ with respect to$x^2 + y^2 = a^2$.
12. What is the polar equation of a conic?
13. Prove that if $(ll_1 + mm_1)a^2$= nn_1, then the lines $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$ are conjugate with respect to $x^2 + y^2 = a^2$.
14. Prove that the polar of a point (x_1, y_1) with respect to the parabola y^2 = 4*ax* is yy_1= 2*a*(x+x_1).
15. Prove that two tangents can be drawn from a given point to an ellipse.
16. Trace the curve $r = asin\Theta, 0 \leq \Theta \leq \pi$.
17. Find the real and imaginary parts of $\tan(\alpha + \beta i)$.
18. Factorize x^5-1.
19. Solve $x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$.
20. Show that the eigen values of a triangular matrix are its entries on the main diagonal.

(2 x 8 = 16)

PART C

Answer **any five** questions. Each question carries 5 marks

21. Prove that the tangents at the ends of a focal chord of the parabola $y^2 = 4ax$ intersect at right angles on the directrix.
22. Find the condition for two circles to cut one another orthogonally.
23. Find the equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ at a point (x_1, y_1).
24. Prove that $\sin^3(\frac{\theta}{3}) + 3\sin^3(\frac{\theta}{3^3}) + 3^2\sin^3(\frac{\theta}{3^3}) + \ldots = \frac{1}{4}[\theta - \sin\theta]$
25. Factorize $x^7 - 1$.
26. Reduce the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to its normal form hence find its rank.
27. Determine the values of λ for which the system of equations

    $3x + y - \lambda z = 0, 4x - 2y - 3z = 0, \quad 2\lambda x + 4y + \lambda z = 0$ may possess nontrivial solutions.

(5 x 5 = 25)

PART D

Answer **any two** questions. Each question carries 12 marks

28. A normal chord to an ellipse makes an angle of 45^0 with the axis. Prove that the square of its length is equal to $\frac{32a^4b^4}{(a^2+b^2)^3}$.
29. Find the equation of the polar of any point (r_1, θ_1) with respect to the conic $\frac{l}{r} = 1 + e\cos\theta$.
30. Prove that $\sin\theta = \theta\prod_{r=1}^{\infty}\left(1 - \frac{\theta^2}{r^2\pi^2}\right)$.
31. Verify Cayley – Hamilton theorem for the matrix A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

(12 x 2 =24)

****