Reg. No…………………………..………………. Name…………………………………………… **20U530**

B.Sc. DEGREE END SEMESTER EXAMINATION OCT. 2020: JANUARY 2021

SEMESTER – 5: MATHEMATICS (CORE COURSE)

COURSE: 15U5CRMAT07- ABSTRACT ALGEBRA

(Common for Regular 2018 admission and Improvement 2017/ Supplementary 2017/2016/2015/2014 admissions)

Time: Three Hours Max. Marks: 75

PART A

Each Question carries 1 Mark Answer All Questions

1. Prove that inverse element of a group is unique.
2. Check whether the algebraic structure $(\mathbb{Z}_5, +_5)$ defined over the set of positive integers is a semi group or not?
3. Show by an example that every proper subgroup of a non abelian group may be abelian.
4. Prove that every permutation in S_n can be written as a product of at most (n-1) transpositions for n ≥ 2.
5. Find the number of elements in the cyclic group of $\mathbb{Z}_{30}$ generated by 25.
6. Give an example of an abelian group which is not cyclic.
7. "Every isomorphism is also a homomorphism" True or False.
8. Describe all units in the ring $\mathbb{Z}_5$.
9. Define characteristics of ring.
10. Find all the units of a ring $\mathbb{Z}_4$. (1 x 10 =10)

PART B

Each Question carries 2 Marks Answer any Eight

11. How many proper subgroups will be there for a group of order 11? Justify your answer.
12. Prove that every cyclic group is abelian.
13. Show that arbitrary intersection of subgroups is a subgroup.
14. Does there exist an element of order 4 in $\mathbb{Z}_{14}$? Justify your answer.
15. Let p and q be prime numbers. Find the number of generators of the cyclic group $\mathbb{Z}_{pq}$.
16. Give two arguments showing that $\mathbb{Z}_4$ is not isomorphic to the Klein 4 – group.
17. Let (R , +) be an abelian group. Show that (R, +, .) is a ring if we define $ab = 0$ for all $a, b \in R$.
18. Find all units of the ring $\mathbb{Z}_{12}$.
19. If R is a ring with unity and N is an ideal of R containing a unit, show that N = R.
20. Determine all ideals of $\mathbb{Z} \times \mathbb{Z}$. (2 x 8 = 16)

PART C

Each Question carries 5 Marks Answer Any Five

21. Construct two different types of group structures of order 4.

22. Prove that every permutation of a finite set is a product of disjoint cycles.
23. State and prove Langrange's theorem.
24. Define an isomorphism of a group. Show that all automorphisms of a group G form a group under function composition.
25. Let $R = \left\{ \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} a, b \in \mathbb{C} \right\}$, where ' $\bar{a}$' is a conjugate of the complex number $'a'$. Prove that R is a non-commutative ring with unity.
26. Show that the rings $2\mathbb{Z}$ and $3\mathbb{Z}$ are not isomorphic.
27. Prove that the characteristics of an integral domain is either a prime number or zero.

(5 x 5 =25)

PART D

Each Question carries 12 Marks Answer Any Two

28. If $A = \{1,2,3\}$ list out all the permutations of A and verify that these permutations form a group under the operation composition.
29. State and prove Cayley's theorem.
30. Define a maximal normal subgroup of a group G. Prove that a homomorphism ϕ of a group G is a one to one function if and only if the kernel of ϕ is $\{e\}$.
31. (a) If a is an integer relatively prime to n then prove that $a^{\phi(n)} \equiv 1(modn)$.

    (b) Find the remainder of 7^{1000} when divided by 24.

(12 x 2 =24)

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