Reg. No………………………………………….. Name…………………………………………… **20U518**

B. Sc. DEGREE END SEMESTER EXAMINATION OCT. 2020: JANUARY 2021

SEMESTER **– 5: MATHEMATICS (CORE COURSE FOR MATHEMATICS & COMPUTER APPLICATIONS)**

COURSE: **15U5CRMAT6-15U5CRCMT6, DIFFERENTIAL EQUATIONS**

(Common for Regular 2018 admission and Improvement / Supplementary 2017/2016/2015 admissions)

Time: Three Hours Max. Marks: 75

PART A

Answer all questions. Each question carries 1 mark

1. Verify for exactness and solve the differential equation $(2xlogy)dx + \left[\frac{x^2}{y} + 3y^2\right]dy = 0$.
2. Prove that if f and g are two different solutions of $\frac{dy}{dx} + P(x)y = Q(x)$ then $f - g$ is a solution of the equation $\frac{dy}{dx} + P(x)y = 0$.
3. Find the integrating factor of the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$.
4. Consider the differential equation $\frac{d^{2y}}{dx^2} - 2\frac{dy}{dx} + y = 0$, show that $e^x$and $xe^x$are linearly independent solutions of the given differential equation on the interval $-\infty < x < \infty$.
5. Solve the differential equation $y^{"} + 2y' + y = 0$.
6. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}cosx$
7. Use operator method to find the solution of the linear system of equations $\frac{dx}{dt} - y = t$, $\frac{dy}{dt} + x = 1$.
8. State the orthogonality of Legender polynomials.
9. Form the PDE by eliminating the arbitrary constants a and b from the following equation $z = (x^2 + a)(y^2 + b)$.
10. Solve $zp + x = 0$. (1 x 10 =10)

PART B

Answer any eight questions. Each question carries 2 marks.

11. Determine whether or not the following Differential equation is exact $(3y + 4xy^2)dx + (2x + 3x^2y)dy = 0$.
12. Solve the equation $(x^2 - 3y^2)dx + 2xydy = 0$
13. Find the orthogonal trajectories of the family of parabolas $y = cx^2$.
14. Solve the initial value problem $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0, y(0) = 2, y'(0) = 6$.
15. Explain the method of variation of parameters to get the Particular integral of a second order linear differential equation.
16. Check whether the following functions $sin^2x, cos^2x, cos2x$ are linearly independent or dependent on the half plane $x \geq 0$.

17. Locate and classify the singular points of the differential equation $(x^2 - 3x)\frac{d^2y}{dx^2} + (x+2)\frac{dy}{dx} + y = 0$.

18. Show that $J_0(kx)$,where k is a constant, satisfies the differential equation $e^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + k^2xy = \quad 0$.

19. Solve $p + q = x + y + z$.

20. Verify that $z = f(x^2 + y^2)$ is a solution of $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$.

(2 x 8 = 16)

PART C

Answer any five questions. Each question carries 5 marks.

21. *Solve* $\frac{dy}{dx} = tan^2(x+y)$.

22. Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$.

23. Solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ by method of variation of parameters.

24. Convert the equation $x^3\frac{d^3y}{dx^3} + 2\,x^2\frac{d^2y}{dx^2} + 2y = \cos(logx)$ into an ordinary differential equation with constant coefficient.

25. Solve $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (3x+2)y = 0$ using power series method.

26. Solve the system of equations $\frac{dx}{dt} = 3x - 4y, \frac{dy}{dt} = x - y$.

27. Solve $(x^2 + y^2 + yz)p + (x^2 + y^2 - xz)q = z(x+y)$. (5 x 5 = 25)

PART D

Answer any two questions. Each question carries 12 marks.

28. Find the value of K such that the parabolas $y = c_1x^2 + K$ are the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 - y = c_2$.

29. Solve $(D^2 + 4D + 4)y = \frac{e^{-2x}}{x^2}$.

30. Prove that $\int_0^\infty e^{-ax}J_0(bx)dx = \frac{1}{\sqrt{a^2+b^2}},\ a > 0$.

31. (i) Solve $y^2p - xyq = x(z - 2y)$

    (ii)Form the PDE by eliminating the arbitrary function from the equation $z = x + y + f(xy)$.

(12 x 2 = 24)

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