Reg. No…………………………………………….   Name………………………………………………   **20U506**

B Sc DEGREE END SEMESTER EXAMINATION OCT. 2020: JANUARY 2021

SEMESTER **– 5: MATHEMATICS (CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATION)**

COURSE: **15U5CRMAT5-15U5CRCMT5, MATHEMATICAL ANALYSIS**

(Common for Regular 2018 admission and Improvement 2017/ Supplementary 2017/2016/2015 admissions)

Time: Three Hours   Max. Marks: 75

PART A

Answer all questions. Each question carries 1 mark.

1. Define deleted neighborhood of a point with an example.
2. Obtain the derived set of $\{x \in \mathbb{Q} : 0 < x < 1\}$
3. State true or false: Every Subset of an uncountable set is uncountable.
4. Define infimum of a set with an example.
5. State true or false: Every open interval (a, b) contains infinitely many rational numbers.
6. Define monotonic sequence with an example.
7. Define Cauchy sequence.
8. Define limit inferior and limit superior of a sequence.
9. State $de\ Moivre's$ Formula.
10. What is the principal argument of $-1 - i$.   (1 x 10 = 10)

PART B

Answer any eight questions. Each question carries 2 marks.

11. Prove or disprove: Set of rational numbers is uncountable.
12. Prove: Closure of a bounded set is bounded.
13. Give an example of a set which is neither open nor closed. Justify your claim.
14. Prove: For any positive real number a there exist a positive real number n such that $n > a$.
15. Prove: The greatest element of a set, if it exists, is the supremum of the set.
16. Prove: In a bounded sequence, limit inferior is the smallest limit point and limit superior is the greatest limit point.
17. Show that the sequence $\{n + (-1)^n n\}$ oscillates finitely.
18. Prove: The sequence $\{S_n\}$ where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ cannot converge.
19. Prove or disprove: $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$.
20. Evaluate $\left(\sqrt{3} + i\right)^7$.   (2 x 8 = 16)

PART C

Answer any five questions. Each question carries 5 marks.

21. Show that interior of a set is an open set.
22. Prove: The union of two closed sets is closed.
23. State and prove: Sandwich theorem.

24. Prove: Every open interval contains a rational number.
25. Show that the sequence $\left\{(b_n)^{1/n}\right\}$ is convergent and find its limit where $b_n = \frac{n^n}{(n+1)(n+2)\dots(n+n)}$.
26. Prove: A necessary and sufficient condition for the convergence of a sequence is that it is bounded and has a unique limit point.
27. Sketch the following sets and determine which are bounded:
    (a) $|z - 2 + i| \leq 1$
    (b) $|2z + 3| > 4$
    (c) $Im(z) > 1$ (5 x 5 =25)

PART D

Answer any two questions. Each question carries 12 marks.

28. Prove the following:
    (a) The derived set of a set is closed.
    (b)The derived set of a bounded set is bounded.
    (c) The supremum of a bounded non-empty set, when not a member of the set, will be a limit point of the set.
29. State the two forms of Completeness Property of real numbers and prove their equivalence.
30. State and prove:
    (a) Cesaro's Theorem
    (b) Cauchy's second theorem on limits
    (c) Cauchy's General Principle of convergence.
31. (a) State and prove the triangular inequality for complex numbers.
    (b) Prove by induction that $\overline{z_1 + z_2 + \cdots + z_n} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}$ and $\overline{z_1 z_2 \dots z_n} = \overline{z_1}\ \overline{z_2} \dots \overline{z_n}$
    (12 x 2 =24)

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