Q. Code:U223 Reg. No………………………….. Name: ………………………………………...……………

B SC DEGREE END SEMESTER EXAMINATION APRIL 2015

SEMESTER - 2: SUBJECT- MATHEMATICS (CORE)

COURSE CODE: U2CRMAT2 - ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

Time: 3 Hrs. Maximum Marks: 75

PART A

Answer **all** *questions.Each question carries* **1** *mark*

1. Write the equation of the tangent to the ellipse at the point (x_1, y_1).
2. Write the condition that two diameters of an ellipse may be conjugate.
3. Represent parametrically the condition of a point on the rectangular hyperbola $xy = c^2$.
4. Find the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
5. Write the polar equation of a parabola.
6. Prove that $\sin(ix) = i\, sinhx$.
7. Separate $\sin(\alpha + i\beta)$ into real and imaginary parts.
8. Prove that $cosh^2x - sinh^2x = 1$.
9. Define rank of a matrix.
10. Write the necessary and sufficient condition for the system $AX = B$ is consistent.

(10 x 1 = 10)

PART B

Answer any **eight** *questions. Each question carries* **2** *marks.*

11. Find the condition for the line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$.
12. Chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are drawn through a fixed point (h, k). Prove that the locus of their middle points is another hyperbola.
13. Show that the sum of the squares of two conjugate semi diameters of an ellipse is constant.

14. Find the equation of the normal at a point whose vertical angle is α on $\frac{l}{r} = 1 + e\ cos\theta$.

15. If x is real, show that $sinh^{-1}x = \log[x + \sqrt{(x^2 + 1)}]$.

16. Separate $\log(\alpha + i\beta)$ into real and imaginary parts.

17. Find the linear factor of $x^8 + 1$.

18. Find the rank of the matrix A=$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$ by reducing into its normal form.

19. Find the eigen values of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

20. Find the inverse of A=$\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ using Cayley Hamilton theorem.

(8 x 2 = 16)

PART C

Answer any **five** *questions. Each question carries* **5** *marks.*

21. Show that the locus of midpoints of chords of a parabola which subtend a right angle at the vertex is another parabola of half the latus rectum of the original parabola.

22. If e and e_1 are the eccentricities of a hyperbola and its conjugate then show that $\frac{1}{e^2} + \frac{1}{{e_1}^2} = 1$.

23. Find the equation of the chord of the conic $\frac{l}{r} = 1 + e\ cos\theta$ joining the points whose vectorial angles are $\alpha - \beta$ and $\alpha + \beta$.

24. If $\sin(A + iB) = x + iy$, show that (a)$\frac{x^2}{cosh^2 B} + \frac{y^2}{sinh^2 B} = 1$ (b) $\frac{x^2}{sin^2 A} - \frac{y^2}{cos^2 A} = 1$.

25. Find the sum of the following series to infinity,

$$cosx\ sinx + \frac{1}{2!}\ cos^2x\ sin2x + \frac{1}{3!} cos^3x\ sin3x + \cdots . \infty.$$

26. Test for consistency and then solve : $x + 2y + z = 2, \quad 3x + y - 2z = 1,$

$4x - 3y - z = 3, \quad 2x + 4y + 2z = 4.$

27. Find the characteristic equation of the square matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

and verify Cayley- Hamilton theorem.

(5 x 5 = 25)

PART D

Answer any **two** *questions. Each question carries* **12** *marks.*

28. Show that the locus of the poles of normal chords of the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^6}{x^2} + \frac{b^6}{y^2} = (a^2 - b^{2)})^2$.

29. If PSP' is a focal chord of a conic, S is a focus and SL is the semi latus rectum, then prove that $\frac{2}{SL} = \frac{1}{SP} + \frac{1}{SP'}$.

30. Resolve into factors the expression $x^{2n} - 2x^n \, cosn\theta + 1$.

31. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

(2 x 12 = 24)