

B. Sc./ B C A DEGREE END SEMESTER EXAMINATION - OCT 2020 : FEBRUARY 2021**SEMESTER 1 : MATHEMATICS - B SC CA(CORE) / B C A (COMPLEMENTARY)****COURSE : 19U1CRCMT1/19U1CPCMT1 : FOUNDATION OF MATHEMATICS***(For Regular - 2020 Admission and Supplementary/Improvement - 2019 Admission)*

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

- What is the cardinality of the following sets:
a) $\{\{a\}\}$ and b) $\{a\}$
- Find $f(S)$ if $f(x) = \lfloor x^2/3 \rfloor$ and $S = \{-2, -1, 0, 1, 2, 3\}$.
- Draw the graph of the function $f(x) = \lfloor 2x \rfloor$ from \mathbb{R} to \mathbb{R} .
- Find $\bigcap_{i=1}^n A_i$ and $\bigcup_{i=1}^n A_i$ if $A_i = \{i, i+1, i+2, \dots\}$.
- List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if
a) $a = b$. b) $a + b = 4$.
c) $a > b$. d) $a \mid b$.
e) $\gcd(a, b) = 1$. f) $\text{lcm}(a, b) = 2$.
- Let R be the relation $R = \{(a, b) : a \text{ divides } b\}$ on the set of integers.
Find (a) R^{-1} (b) R
- Suppose that the function f from A to B is a one-to-one correspondence. Let R be the relation that equals the graph of f . That is, $R = \{(a, f(a)) : a \in A\}$. What is the inverse relation R^{-1}
- a) Define a partial ordering.
b) Show that the divisibility relation on the set of positive integers is a partial order.
- Draw the Truth Table for the Conditional Statement $p \rightarrow q$.
- Translate the sentence into logical expression: "You will get an A in the class if and only if you either do every exercise in textbook or you get an A on the final".
- Find the g.c.d of the pair of integers 58 and 86 and express it as a linear combination of the two integers.
- Compute $\Phi(873)$

(2 x 10 = 20)**PART B****Answer any 5 (5 marks each)**

- Define composition of functions. Let f and g be the functions from the set of integers to set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of f and g ?
- What are the terms a_0, a_1, a_2, a_3 of the $\{a_n\}$ where $a_n = 2^n + (-2)^n$
- For any sets, Prove that $A - B = A \cap \overline{B}$
- These relations on the set of real numbers:
 $R_1 = \{(a, b) \in \mathbb{R}^2 \mid a > b\}$, the "greater than" relation,
 $R_2 = \{(a, b) \in \mathbb{R}^2 \mid a \geq b\}$, the "greater than or equal to" relation,
 $R_3 = \{(a, b) \in \mathbb{R}^2 \mid a < b\}$, the "less than" relation,
 $R_4 = \{(a, b) \in \mathbb{R}^2 \mid a \leq b\}$, the "less than or equal to" relation,
 $R_5 = \{(a, b) \in \mathbb{R}^2 \mid a = b\}$, the "equal to" relation,
 $R_6 = \{(a, b) \in \mathbb{R}^2 \mid a \neq b\}$, the "unequal to" relation.

Find

- a) $R1 \cup R3$. b) $R1 \cup R5$.
c) $R2 \cap R4$. d) $R3 \cap R5$.
e) $R1 - R2$. f) $R2 - R1$.
g) $R1 \oplus R3$. h) $R2 \oplus R4$.

17. Show that the “divides” relation is the set of positive integers in not an equivalence relation.
18. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.
19. Show that “If n is an odd integer, then n^2 is odd.” By direct proof.
20. Find the number and the sum of the divisors of 4116.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. a) Show that the set of all positive rational numbers is countable.
b) Show that the set of all real numbers is uncountable
22. Draw the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$.
23. Show that these statements about the integer n are equivalent:
p1: n is even.
p2: $n-1$ is odd.
p3: n^2 is even.
24. Prove that $18! + 1 \equiv 0 \pmod{23}$.

(10 x 3 = 30)