B. Sc DEGREE END SEMESTER EXAMINATION - OCT. 2020 : FEBRUARY 2021
SEMESTER 1 : MATHEMATICS
COURSE : **19U1CRMAT01 : CALCULUS**
(For Regular - 2020 Admission & Improvement / Supplementary 2019 Admission)

Time : Three Hours   Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Find the open intervals on which $f(x) = x^2 - 3x + 8$ is decreasing

2. Define a decreasing function

3. Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing

4. Obtain the point of inflection of $f(x) = xe^{-x}$

5. Evaluate $\lim\limits_{x \to +\infty} \dfrac{x^{-4/3}}{\sin\left(\frac{1}{x}\right)}$

6. Evaluate $\lim\limits_{x \to 0} (1 + \sin x)^{1/x}$.

7. Find $\lim\limits_{x \to \frac{\pi}{2}} (\cos x) \tan x$.

8. Find the formula for the volume of a sphere of radius r.

9. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y axis.

10. Find the area of the region enclosed by the curves $y = e^x$, $y = e^{2x}$, $x = 0$ and $x = \ln 2$.

11. Describe the domain of $f(x, y, z) = e^{xyz}$.

12. Find f_{xxy} for $f(x, y) = y^2 e^x + y$.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Locate the critical points and identify which critical points are stationary $f(x) = 4x^4 - 16x^2 + 17$

14. Show that among all rectangles with perimeter p, the square has the maximum perimeter.

15. Verify that the hypotheses of the mean value theorem satisfied on the given interval, and find all values of c in that interval that satisfy the conclusion of the theorem $f(x) = x^2 - x; [-3, 5]$.

16. Determine $\lim\limits_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx}$

17. Find the volume of the solid that results when the region enclosed by the curve $y = 9 - x^2$, $y = 0$ is revolved about x-axis.

18. Use cylindrical shells to find the volume of the solid generated when the region R under $y = x^2$ over the interval $[0, 2]$ is revolved about the line $y = -1$.

19. Use cylindrical shells to find the volume of the solid generated when the region enclosed by the curve $y = \sqrt{x}$, $x = 4$, $x = 9$, $y = 0$ is revolved about y-axis.

20. Use a chain rule to find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ at $(1, -2)$ if $f(x, y) = x^2 y^2 - x + 2y$, $x = \sqrt{u}$, $y = uv^3$.

(5 x 5 = 25)

21. Sketch the graph of the curve $y = x + \dfrac{1}{x}$.

22. Prove that $\sin h^{-1}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$.

23. Use cylindrical shells to find the volume of the solid generated by the revolving about y-axis the region enclosed by $y = 2x - 1, y = -2x + 3, x = 2$.

24. Let $w = 4x^2 + 4y^2 + z^2, x = p\sin\phi\cos\theta, y = p\sin\phi\sin\theta, z = p\cos\phi$. Find $\dfrac{\partial w}{\partial p}, \dfrac{\partial w}{\partial\phi}, \dfrac{\partial w}{\partial\theta}$.

(10 x 3 = 30)