SACRED HEART COLLEGE (AUTONOMOUS)
THEVARA,KOCHI-13
(Affiliated to Mahatma Gandhi University, Kottayam)

MA/MSc/M Com DEGREE EXAMINATION 2014 – 15
FIRST SEMESTER
SUBJECT : MATHEMATICS
COURSE : P1MATT04 : GRAPH THEORY

Time : 3 Hours Max. Marks : 75

PART A

(Answer any five questions.
Each question carries 2 marks.)

1. Define the line graph of a graph. Draw the line graph of $K_{2,2}$.
2. If the degree of each vertex of a graph G with n vertices and m edges is either k or $k+1$, then show that the number of vertices of degree k is $(k+1)n-2m$.
3. Show that every tree with atleast two vertices contains atleast two pendant vertices.
4. If u and v are non-adjacent vertices of a tree T, then prove that $T+uv$ contains a unique cycle.
5. Define the covering number of a graph and find it for the wheel graph W_5.
6. Prove that a simple k-regular graph on $2k-1$ vertices is Hamiltonian.
7. State the Heawood five color theorem.
8. Show that the Petersen graph, P is non-planar.

PART B

(Answer any five questions.
Each question carries 5 marks.)

9. If G is a simple graph, then show that G or G^c is connected.
10. If $(d_1, d_2 \ldots d_n)$ is the degree sequence of a graph and r is any positive integer, then show that $\sum_{i=1}^{n} d_i^r$ is an even number.
11. Prove that every tree has a center consisting of either a single vertex or a pair of adjacent vertices.
12. Explain Kruskal's algorithm with a suitable example.
13. Prove that if G is k-critical, then $\delta(G) \geq k-1$. Deduce that for any graph G, $\chi(G) \leq 1 + \triangle(G)$.

14. Show that in a critical graph G, no vertex cut is a clique.

15. Show that the graph K_5 is non-planar.

16. Prove that every planar graph is 6-vertex-colorable.

PART C

(Answer either Part I or Part II of each question. Each question carries 10 marks.)

17. (I) (a) Show that every tournament contains a directed Hamiltonian path.
 (b) Prove that a connected graph G with atleast two vertices contains atleast two vertices that are not cut vertices.

 (II) (a) Show that a simple graph is bipartite if and only if it has no odd cycles.
 (b) If a graph G is simple and $\delta \geq \frac{n-1}{2}$, then prove that G is connected. Also draw a non-simple disconnected graph with $\delta \geq \frac{n-1}{2}$.

18. (I) State and prove Cayley's theorem.

 (II) (a) Prove that a simple graph is a tree if and only if every pair of vertices are connected by a unique path.
 (b) Show that a simple connected graph with n vertices is a tree if and only if it has exactly $n-1$ edges.

19. (I) For a non-trivial connected graph G, prove that the following statements are equivalent.
 (a) G is Eulerian.
 (b) The degree of each vertex of G is an even positive integer.
 (c) G is an edge-disjoint union of cycles.

 (II) Show that for every positive integer k, there exists a triangle free graph with chromatic number k.

20. (I) State and prove König's theorem. Is the converse true? Justify.

 (II) (a) State and prove Euler's formula for a connected plane graph.
 (b) Prove that a simple planar graph with minimum degree atleast five contains atleast twelve vertices. Also draw a simple plane graph on twelve vertices with minimum degree five.