Reg. No .... Name **20P1044**

M. Sc DEGREE END SEMESTER EXAMINATION - OCT 2020 : FEBRUARY 2021

SEMESTER 1 : MATHEMATICS

COURSE : 16P1MATT04 : ORDINARY DIFFERENTIAL EQUATIONS

(For Regular - 2020 Admission and Supplementary - 2016/2017/2018/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer All (1.5 marks each)

1. Find any one characteristic vector of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$.
2. Transform the linear system $t\frac{dx}{dt} = ax + by$ into a linear system with constat coefficients.
   $t\frac{dy}{dt} = cx + dy$
3. Does there exist any homogeneous linear system of two unknown functions on an interval $0 \leq t \leq 2\pi$ such that its wronskian of two solutions is W(t) = cos(t) on $0 \leq t \leq 2\pi$. Justify your answer.
4. Is $x_0 = 0$ a regular singular point of the equation $y'' + \frac{2}{x}y' - \frac{2}{x^2}y = 0$. Justify your answer
5. Write Gauss's Hypergeometric equation.
6. Define characteristic values and characteristic functions of a Strum - Liouville problem.
7. The sequence of functions $\{sin(nx)\}_{n=1}^{\infty}$ is orthonormalized with respect to the weight function $r(x) = 1$ on the interval $0 \leq x \leq \pi$. State true or false and justify your answer.
8. Show that $L[x] = \frac{1}{p^2}$.
9. Find a function f whose Laplace transform is $\frac{2}{p+3}$.
10. Find the Laplace transform of xe^x.

(1.5 x 10 = 15)

PART B

Answer any 4 (5 marks each)

11. Consider the vector functions $\varphi(t) = \begin{bmatrix} t \\ 1 \end{bmatrix}$ and $\psi(t) = \begin{bmatrix} te^t \\ e^t \end{bmatrix}$.Show that the constant vectors $\varphi(t_0)$ and $\psi(t_0)$ are linearly dependent for each t_0 in the interval $0 \leq t \leq 1$, but the vector functions φ and ψ are linearly independent on $0 \leq t \leq 1$.
12. Find the general solution of the system $\frac{dx}{dt} = 5x - y, \frac{dy}{dt} = 3x + y$.
13. Find the indicial equation and its roots of the differential equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.
14. Find characteristic values and characteristic functions of the Strum - Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(L) = 0$, where $L > 0$.
15. Find $L[sin^2(ax)]$ and $L[cos^2(ax)]$ without integrating. How are these two transforms related to one another?
16. Find $L^{-1}[\frac{1}{(p^2+a^2)^2}]$ by using convolution.

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. Solve $t\frac{dx}{dt} = 2x + 3y$

$t\frac{dy}{dt} = 2x + y$

OR

2. Find all characteristic values and vectors of the matrix $A = \begin{bmatrix} 1 & 3 & -6 \\ 0 & 2 & 2 \\ 0 & -1 & 5 \end{bmatrix}$

18.1. The equation $x^2y'' - 3xy' + (4x+4)y = 0$ has only one Frobenius series solution. Find it.

OR

2. Find two independent Frobenius series solution of $x^2y'' - x^2y' + (x^2-2)y = 0$

19.1. Consider the Strum-Liouville problem $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + [q(x) + \lambda r(x)]\,y = 0$ with boundary conditions $A_1y(a) + A_2y'(a) = 0$ and $B_1y(b) + B_2y'(b) = 0$ where A_1, A_2, B_1, B_2 are real constants such that A_1 and A_2 are not both zero and B_1 and B_2 are not both zero. Show that the characteristic funcions corresponding to distinct characteristic values are orthogonal with respect to the weight function $r(x)$ on the interval $a \leq x \leq b$.

OR

2. Find characteristic values and characteristic functions of the Strum - Liouville problem $\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{\lambda}{x}y = 0, y(1) = 0, y'(e^{\pi}) = 0.$

20.1. If f is periodic with period a, then show that $F(p) = \frac{1}{1-e^{-ap}}\int_0^a e^{-px}f(x)dx.$

OR

2. Use the principle of superposition to solve the equation $y'' + 5y' + 6y = 5e^{3t}$ with initial conditions y(0)=0, y'(0)=0.

(10 x 4 = 40)