Reg. No .... Name **20P1017**

M. Sc DEGREE END SEMESTER EXAMINATION - OCT 2020 : FEBRUARY 2021

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT02 ; BASIC TOPOLOGY**

(For Regular - 2020 Admission and Supplementary 2019/2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer All (1.5 marks each)

1. Is $\{3\}$ open in the usual topology on $\mathscr{R}$? Justify.
2. Define relative topology. Is openness a hereditary property?
3. Consider $A = [0, 1)$ as a subset of R with usual topology. Find int(A) and cl(A)?
4. Consider R with usual topology. Check whether R is Lindeloff.
5. Define 1) Embedding 2) i^{th} projection and 3) Continuity.
6. Define a compact set A in a space $(X, \mathscr{T})$. Give an example of a set that is not compact.
7. Prove that every non empty connected subset is contained in a unique component.
8. Consider R with usual topology. Check whether $N \subset R$ is connected.
9. Define (i) Tychnoff space (ii) Regular Space.
10. Define (i) T_1 space (ii) Normal Space.

(1.5 x 10 = 15)

PART B

Answer any 4 (5 marks each)

11. If a space is second countable then prove that every open cover of it has a countable subcover.
12. Show that closed subspace of compact space is compact.
13. Show that every open surjective map is a quotient map.
14. Prove that if X is locally connected then components of open subsets of X are open.
15. Differentiate connectedness and locally connectedness with an example.
16. Prove that the real line with semi open interval topology is normal.

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. Prove that the usual topology in the Euclidean plane R^2 is strictly weaker than the topology induced on it by the lexicographic ordering.

OR

2. (a) A sequence in a co finite space is convergent if and only if there exist 1 term which repeats infinitely many times.
(b) Define closure $\overline{A}$ and derived set $A^{'}$ of a subset A of a space X and show that $\overline{A} = A \cup A^{'}$.

18.1. (a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.(b) Prove that continuous image of a compact space is compact.

OR

2. Let $[(X_i, \mathscr{T}_i), i = 1, 2, \dots n]$ be a collection of topological spaces and $(X, \mathscr{T})$ their topological product. Prove that each projection π_i is continuous. Also show that if Z is any space then the fuction $f : Z \to X$ is continuous if and only if $\pi_i o f : Z \to X_i$ is continuous for all $i = 1, 2, \dots n$.

19.1. (a) Prove that every closed and bounded interval is compact. (b) Show that union of collection of connected subsets of X having a common point is connected.

OR

2. (a)Establish five equivalent condition for a space to be connected.(b) Prove that every space is a disjoint unioin of its components.

20.1. (a)Show that the axioms T_0, T_1, T_2, T_3 and T_4 form a hierarchy of progressively stronger condition.
(b) Every continuous, one to one function from a compact space onto a Hausdorff space is an embedding.

OR

2. (a) Show that every map from a compact space into a T_2 space is closed.
(b) Let X be a completely regular space. Suppose F is a compact subset of X, C is a closed subset of X and $F \cap C = \phi$. Prove that there exist a continuous function from X into the unit interval which takes the value 0 at all points of F and the value 1 at all points of C.

(10 x 4 = 40)