Reg. No .... Name **20P3004**

MSc DEGREE END SEMESTER EXAMINATION - OCT/NOV 2020: JAN 2021

SEMESTER 3: MATHEMATICS

COURSE : **16P3MATT11: PARTIAL DIFFERENTIAL EQUATIONS**

(For Regular - 2019 Admission and Supplementary 2016/2017/2018 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer all (1.5 marks each)

1. Derive a partial differential equation from $x^2+y^2+(z-c)^2=a^2$
2. Verify that the differential equation $yz\,dx+(x^2y-zx)\,dy+(x^2z-xy)\,dz=0$ is integrable
3. Derive a partial differential equation from $x^2+y^2=(z-c)^2\tan^2\alpha$
4. Find the complete integral of the equation $p^2y(1+x^2)=qx^2$
5. Find the complete integral of the equation $p^2-q^2=z$
6. Solve $\dfrac{\partial^4 z}{\partial x^4}+\dfrac{\partial^4 z}{\partial y^4}=\dfrac{2\partial^4 z}{\partial x^2\partial y^2}$
7. Find the particular integral of $(D^2-D')z=2y-x^2$
8. Find the particular integral of $r+3s+2t=x+y$
9. Write the Laplace's equation
10. Define equipotential surface

(1.5 x 10 = 15)

PART B

Answer any 4 (5 marks each)

11. Find the general solution of the equation $px(x+y)=qy(x+y)-(x-y)(2x+2y+z)$
12. Find the orthogonal trajectory on the conicoid $(x+y)z=1$ of the conics in which it is cut by the system of planes $x-y+z=k$ where k is the parameter
13. Find the complete integral of the equation $z^2=pqxy$
14. If u is the complementary function and z_1 is particular integral of a linear pde $F(D,D')z=f(x,y)$. Then prove that $u+z_1$ is a general solution of the equation.
15. Let $\alpha_r D+\beta_r D'+\gamma$ is a factor of $F(D,D')$ and $\phi_r(\xi)$ is an arbitrary function of the variable ξ. Prove that if $\alpha_r\neq 0, u_r=\exp(\frac{-\gamma_r x}{\alpha_r})\phi_r(\beta_r x-\alpha_r y)$ is a solution of the equation $F(D,D')z=0$.
16. Show that if a function z satisfies the differential equation $\frac{\partial^2 z}{\partial x^2}\frac{\partial z}{\partial y}=\frac{\partial^2 z}{\partial x\partial y}\frac{\partial z}{\partial x}$ it is of the form $f(x+g(y))$, where f and g are arbitrary

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. Prove that the Pfaffian differential equation $\bar{X} \cdot d\bar{r} = 0$ is integrable if and only if $\bar{X} \cdot$ curl$\bar{X} = 0$

OR

2. Find the surface which intersect the surface of system $z(x+y) = c(3z+1)$ orthogonally and which passes through the surface $x^2 + y^2 = 1,\ z = 1$

18.1. Explain Charpit's method of solving the pde $f(x, y, p, q) = 0$ and solve $p = (z + qy)^2$

OR

2. Show that the integral surface of the equation $2y(1+p^2) = pq$ which is circumscribe about the cone $x^2 + z^2 = y^2$ has the equation $z^2 = y^2(4y^2 + 4x + 1)$

19.1. (i) Solve $(D^2 - D')z = e^{x+y}$
(ii) Solve $(r + s - 2t) = e^{x+y}$

OR

2. (i) Solve $r + 3s + 2t = x + y$
(ii) Solve $(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = e^{x+y}$

20.1. Describe Monge's method. Solve the equation $r = t$ the wave equation using Monge's method

OR

2. Describe Monge's method. Solve $z^2(rt - s^2) + z(1+q^2)r - 2pqzs + z(1+p^2)t + 1 + p^2 + q^2 \quad = 0$ using Monge's method

(10 x 4 = 40)