Reg. No…………………………………………….   Name……………………………………………   **20U642**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER **– 6: MATHEMATICS (CORE COURSE)**

COURSE: **15U6CRMAT12: LINEAR ALGEBRA AND METRIC SPACES**

(Common for Regular 2017 Admission & Supplementary 2016 /2015/2014 Admissions)

Time: Three Hours   Max. Marks: 75

PART A

Answer all questions. Each question carries 1 mark

1. Define linear independence of vectors.
2. What is the dimension of the space of all polynomials in one variable over the field of real numbers?
3. If a nonzero vector space is spanned by 5 vectors what can you say about its dimension?
4. Show that T: R → R defined by T(x) = 2x is linear.
5. Linear transformation T: $R^2 \to R^2$ is such that T(0,1) = (0,0) and T(1,0) = (0,2) .Find T(x, y).
6. Define the null space of a linear transformation.
7. What is the usual metric on R?
8. What do you mean by an interior point of a metric space?
9. Define the convergence of a sequence in a metric space.
10. What is a dense set? Give an example. (1 x 10 = 10)

PART B

Answer any Eight questions. Each question carries 2 marks

11. Show that a set containing the zero of a vector space is linearly dependent.
12. If an $n\times n$ matrix has two identical rows, what can be concluded about its rank?
13. Check whether { (1, 0,-1), (0,0, 2), (1,0,0)} is a basis of R^3.
14. Give an example of a nonzero linear transformation with nonzero null space.
15. Prove that a subset of a vector space consisting of a single vector 'v' is linearly dependent if and only if $v = 0$.
16. Define the rank and nullity of a linear transformation.
17. What is an open subset of a metric space?
18. Show that a finite subset is a closed subset of a metric space.
19. Is the set of natural numbers $\boldsymbol{N}$ is open in the metric space $\boldsymbol{R}$ of real numbers. Justify
20. Give an example of a sequence with more than one limit point. (2 x 8 = 16)

PART C

Answer any Five questions. Each question carries 5 marks

21. Prove that the set of all polynomials in one variable is a subspace of the space of all functions from R into R.
22. Find the dimension of the space of all 2×2 matrices by establishing a basis.
23. Let V be the vector space of all polynomials of degree at most three. Let $T: V \rightarrow V$ be the linear transformation given by $T\big(p(x)\big) = p^{|}(x)$ where $p^{|}(x)$ is the derivative of $p(x)$. Find the matrix of the linear transformation T relative to the basis$\{1, x, x^2, x^3\}$.
24. Let $T : R^2 \rightarrow R^2$ be defined by T(x, y) = (y, x). Prove that T is linear. What is the null space of T? Is T invertible?
25. Prove that in any metric space X each open sphere is an open set. What about the converse. Justify.
26. Let X be a metric space and G be open in X. Prove that G is disjoint from a set A if and only if G is disjoint from $\overline{A}$.
27. Show that the Cantor set is nowhere dense. (5 x 5 = 25)

PART D

Answer any Two questions. Each question carries 12 marks

28. (a) Let V be a vector space over R. Suppose that there are vectors $v_1, v_2, \ldots v_n$ which span V. Prove that V is finite dimensional.

    (b) Find three vectors in R^3 which are linearly dependent, and are such that any two of them are linearly independent.
29. The linear transformation T on R^3 is defined by T(x, y, z) = (3x+z, -2x+y, -x+2y+4z)

    (a) What is the matrix of T in the standard ordered basis for R^3.

    (b) What is the matrix of T in the basis { (1, 0, 1), (-1, 2, 1), (2, 1, 1)}
30. Define $d(X, Y) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$ where $X = (x_1, y_1)$ and $Y = (x_2, y_2)$. Show that d is metric on R^2. Draw the closed sphere of radius one unit and center at the origin.
31. (a) State and prove Cantor's Intersection Theorem.

    (b) Let X and Y be metric spaces and f be function from X into Y. Prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y. (12 x 2 = 24)

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