Reg. No.. Name.. **20U604**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2020

SEMESTER – 6: MATHEMATICS (Common for Mathematics / Computer Applications)

COURSE: 15U6CRMAT9/15U6CRCMT7: REAL ANALYSIS

(Common for Regular - 2017 Admission / Improvement 2016/ Supplementary 2016/ 2015/2014 Admissions)

Time: Three Hours | Max. Marks: 75

Part A

Answer **all** questions. Each question carries 1 mark.

1. Give an example of a series Σu_n, where $\lim u_n = 0$, but the series is divergent.
2. When the geometric series $1 + r + r^2 + \cdots$ is convergent ?
3. Give an example of a divergent positive term series.
4. Define an alternating series.
5. Give an example of a function with a removable discontinuity.
6. Give an example of a continuous function on $\mathbb{R}$ which does not attain its infimum.
7. Define $\bar{\int} f\, dx$ over $[a, b]$.
8. Define refinement of a partition.
9. What is the pointwise limit of $f_n(x) = \dfrac{nx}{1+n^2x^2}$.
10. State the Cauchy criterion for the uniform convergence of a sequence of functions.

(10×1=10)

Part B

Answer any **eight** questions. Each question carries 2 marks.

11. State the limit form of the comparison test.
12. If Σu_n is a positive term series such that $\lim\limits_{n\to\infty} (u_n)^{\frac{1}{n}} < 1$, what can you say about the convergence of Σu_n?
13. Show that the series $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} + \cdots$ is not convergent.
14. Find the points of discontinuity of the function $f(x) = \dfrac{x - |x|}{x}$.
15. Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.
16. Show that a constant function is Riemann integrable.
17. Show that the function $f(x) = \begin{cases} 0, & \text{if } x \text{ rational} \\ 1, & \text{if } x \text{ irrational} \end{cases}$ is not Riemann integrable.

18. Compute $\int_{-1}^{1} f dx$, where $f(x) = |x|$.

19. Find an interval on which $\{f_n\}$ where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent.

20. Prove that the sequence $f_n(x) = \frac{x}{1+nx^2}$, x being real, converges uniformly on any closed interval I. (8×2=16)

Part C

Answer any **five** questions. Each question carries 5 marks.

21. Prove that every absolutely convergent series is convergent.

22. Prove that if a series Σu_n of positive monotonic decreasing terms converges then not only $u_n \to 0$. but also $nu_n \to 0$ as $n \to \infty$.

23. Test for convergence of the series $\sum \frac{n^2-1}{n^2+1} x^n$, $x > 0$.

24. Prove that if a function f is continuous on $[a, b]$ and $f(a) \neq f(b)$, then it assumes every value between $f(a)$ and $f(b)$.

25. Prove that for any two partitions P_1 and P_2, $L(P_1, f) \leq U(P_2, f)$.

26. Prove that if f is bounded and integrable on $[a, b]$, then $|f|$ is also bounded and integrable on $[a, b]$ and $|\int_a^b f dx| \leq \int_a^b |f| dx$.

27. Show that the sequence $\left\{\frac{nx}{1+n^3x^2}\right\}$ converges uniformly to zero for $0 \leq x \leq 1$. (5×5=25)

Part D

Answer any **two** questions. Each question carries 12 marks.

28. (a) State and prove Cauchy root test.

    (b) Study the convergence of the series $\sum_{n=0}^{\infty} \frac{3^n}{n!}$.

29. (a) Prove that if a function f is continuous on a closed interval $[a, b]$, then it attains its bounds at least once in $[a, b]$.

    (b) Prove that if f is continuous on $[a, b]$ and $f(x) \in [a, b]$ for every $x \in [a, b]$, then f has a fixed point.

30. (a) State and prove a necessary and sufficient condition for integrability of a bounded function.

    (b) Prove that every continuous function is integrable.

31. (a) Show that the sequence $\left\{\frac{x}{n+x}\right\}$ is uniformly convergent in $[0, k]$, $k < \infty$ but only pointwise convergent when the interval extends to ∞.

    (b) Discuss the convergence of the sequence of functions $\{x^n\}$ on $[0, 1]$. (2×12=24)