

**B. Sc. DEGREE END SEMESTER EXAMINATION OCTOBER 2018****SEMESTER – 3: STATISTICS (CORE COURSE)****COURSE: 15U3CRCST3-15U3CPSTA3, PROBABILITY DISTRIBUTIONS***(For Regular - 2017 Admission and Supplementary / Improvement 2016 & 2015 Admissions)*

Time: Three Hours

Max. Marks: 75

*Use of Scientific calculators and Statistical tables permitted***PART A**Answer **all** the questions, each question carries one mark

1. Define raw and central moments.
2. Give any one method for deriving raw moments from moment generating function.
3. Let  $X$  denotes the number of white balls when 6 balls are drawn with replacement from a bag containing 6 white and 4 red balls. Give the distribution of  $X$  with parameter values.
4. What do you mean by Lack of Memory Property (LMP).
5. Distinguish between Statistic and Parameter.
6. Define Standard error.
7. Define convergence in probability.
8. What is the relation between an exponential distribution and a gamma distribution?
9. If  $X$  follows a normal distribution with mean  $\mu$  and variance  $\sigma^2$ , then  $\mu_6$  is.
10. Define conditional expectation. (1 x 8 = 8)

**PART B**Each question carries **three** marks. Maximum marks from this part is 15

11. State and prove Cauchy Schwartz inequality.
12. State and prove the additive property of Binomial distribution
13. Let  $X$  be a continuous random variable with distribution function  $F(x)$ ,  $x \in R$ . Derive the density function of the random variable  $Y = F(x)$
14. If  $X$  follows a uniform distribution in  $(4, 6)$ , find  $P(|X - 4| < 1)$
15. Let  $X_1, X_2, \dots, X_7$  and  $Y_1, Y_2, \dots, Y_8$  be independent samples from normal populations with means 8, 12 and variances 14 and 16 respectively. Find,  $P[X - \bar{Y} \geq 0]$  Where  $X$  and  $\bar{Y}$  are the sample means.
16. Let  $F$  be a random variable following F-distribution with degree of freedom 8 and 12. Then find the values of  $a$  and  $b$  in the following cases  
(a)  $P(F \leq a) = 0.05$  (b)  $P(F \leq b) = 0.99$
17. Assume  $X$  is a normal random variable with mean 12 and variance 25 such that  $P(X > k_1) = 0.68$  and  $P(k_1 < X < k_2) = 0.32$ . Find the values of the constants  $K_1$  and  $K_2$ .

**PART C**

Each question carries **five** marks. Maximum marks from this part is 20

18. If  $X$  and  $Y$  are independent variables following geometric distribution with parameter  $p$ , find the conditional distribution of  $X$  given  $X + Y$ .

19. Let  $X$  be a random variable with probability density function.

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{Otherwise} \end{cases}$$

Find the moment generating function and hence find the mean and the variance.

20. If  $x_1, x_2, \dots, x_{100}$  is a random sample of size 100 taken from a poisson distribution with mean 4 find  $P(X > 3.8)$  where  $X$  is the sample mean.

21. Obtain the moment generating function of a normal random variable with mean  $\mu$  and variance  $\sigma^2$

22. If  $X \sim N(0,1)$ , show that distribution of  $X^2$  is gamma.

23. Distinguish between beta distribution of the first kind and beta distribution of the second kind.

**PART D**

Each question carries **ten** marks. Maximum marks from this part is 30

24. (a) State and Prove the Tchebycheff's Inequality

(b) Let  $X$  be a discrete random variable assuming values -3, -1, 1, 6, 7 with equal probability. Find the lower bound to  $P(|X - 2| \leq 2)$ . Also find the actual probability.

25. Let the random vector  $(X,Y)$  has the joint density given by  $f(x, y) = \begin{cases} kxy, & 0 < x < y < 2 \\ 0, & \text{Otherwise} \end{cases}$

(a) Find the constant  $k$

(b) Find the Covariance between the variables  $X$  and  $Y$

26. Fit a Poisson distribution to the following data :

$X :$	0	1	2	3	4
$f :$	120	60	12	6	2

27. Explain the important methods of sampling. What are their advantages and disadvantages?

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