Reg. No.................................. Name.................................. U636

BSc DEGREE END SEMESTER EXAMINATION MARCH 2017

SEMESTER **– 6: MATHEMATICS (CORE COURSE)**

COURSE: **U6CRMAT12: LINEAR ALGEBRA AND METRIC SPACES**

(For Regular - 2014 Admission)

Time: Three Hours Max. Marks: 75

PART A

Answer **all** question. Each question carries 1 mark.

1. Give any subspace of the vector space R^2 over R.
2. Is the set {(1, 2), (1, 3), (1, 4)} linearly independent in $\mathbf{R}^2$? Justify.
3. If $V_1 = Span\{(1,1)\}$,and $V_2 = span\{(1,0)\}$ are two subspaces of R^2 over R. Find $(V_1 + V_2)$.
4. Define a linear transformation.
5. Check whether the transformation $T: R^2 \rightarrow R^2$ given by $T(x, y) = (x + a, y + a)$, where $'a' \neq 0$, a constant is **linear or not**.
6. Show that if T is a linear transformation on any vector space V then $T(0) = 0$ where $0 \in V$.
7. Write the indiscrete metric on **C**.
8. Write any two properties of Cantor Set.
9. Give an example of an incomplete metric space.
10. Define a continuous function in a metric space.

(1 x 10 = 10)

PART B

Answer **any eight** questions. Each question carries 2 marks.

11. Is **R** over **Q** is a vector space with usual addition and scalar multiplication? If yes find its dimension.
12. Show that intersection of two subspaces of a vector space is a vector space. Is union of two subspaces is a subspace? Justify.
13. Define basis of a vector space. Show that {(1, 0), (-1, 1)} is a basis for $\mathbf{R}^2$.
14. Let $T: R^3 \rightarrow R^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 - x_2, 0)$. Find the rank and nullity of the linear transformation.
15. Show that sum of two linear transformation from a vector space V in to a vector space W is again a linear transformation.
16. Let T and U be linear transformations from R^2 to R^2 given by $T(x_1, x_2) = (x_2, x_1)$ and $U(x_1, x_2) = (x_1, 0)$. Find TU and UT. Is $TU = UT$?
17. Verify that the map $d(x, y) = ¿ x^2 - y^2 \vee ¿$, for any $x, y \in R$ is a metric on R.
18. Prove that in any metric space X, X and Φ are always open.
19. Define a Cauchy sequence in a metric space X. Say True or False is every Cauchy sequence in a metric space X is convergent. Justify.
20. Consider R with usual metric. Prove that N, the set of all natural numbers is nowhere dense in R.

(2 x 8 = 16)

PART C

Answer **any five** questions. Each question carries 5 marks.

21. i) Let V be the set of pairs (x,y) of real numbers and F be the field of real numbers. Define $(x,y)+(x_1,y_1)=(x+x_1,0)$ and $c(x,y)=(cx,0);c\in F$. Is V with these operations is a Vector space.

    ii) Prove that two vectors are linearly dependent, one of them is a scalar multiple of other.

22. i) Find three vectors in R^3 which are linearly dependent, and are such that any two of them are linearly independent.

    ii) Let V be the vector space of all 2x2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V which has 4 elements.

23. i) Let $T:R^6\to W$ be a linear transformation where R^6,Wvector spaces are over R, also $S=\{Te_2,Te_4,Te_6\}$ spans$W$. Prove that Kernel (T) contains more than one element.

    ii) Give an example of an invertible linear transformation. Prove that it is invertible, and find its inverse.

24. Let V be the vector space of all polynomials of degree at most three. Let $T:V\to V$ be the linear transformation given by $T(p(x))=p¿(x)$ where $p¿(x)$ is the derivative of $p(x)$. Find the matrix of the linear transformation T relative to the basis$\{1,x,x^2,x^3\}$.

25. Prove that in any metric space(X,d), (i) any union of open sets in X is open (ii) Finite intersection of open sets in $X$is open.

26. Let X be a metric space, prove that a subset F of X is closed if and only if its compliment F^c is open.

27. Let *X* and Y be metric spaces and f is a mapping from X into Y. Show that f is continuous at x_0 if and only if $x_n\to x_0$ then $f(x_n)\to f(x_0)$.

(5 x 5 = 25)

PART D

Answer **any two** questions. Each question carries 12 marks

28. i) Let W be the space generated by the polynomials $v_1=t^3-2t^2+4t+1, v_2=2t^3-3t^2+¿$ $9t-1,\quad v_3=t^3+6t-5, v_4=2t^3-5t^2+7t+5$. Find a basis and dimension of W.

    ii) Define row space of a matrix. Show that row equivalent matrices have the same row space.

29. Define the nullity and rank of a linear transformation *T*. For a linear transformation $T:V\to W$,prove that $Rank(T)+Nullity(T)=dimV$ .Hence prove that, if $dimV=dimW$, then *T* is one to one if and only if *T* is onto.

30. i) State and prove Baire's Theorem.

    ii) State and prove Cantor's Intersection Theorem.

31. i) Let X and Y are metric spaces and 'f' is a mapping from X in to Y. Prove that f is continuous if and only if $f^{-1}(G)$ is open in X, Whenever G is open in Y.

ii) Show by an example that under continuous mapping image of an open set need not open.

(12 x 2 = 24)

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