

**B. Sc. DEGREE END SEMESTER EXAMINATION MARCH 2017****SEMESTER - 6: MATHEMATICS (CORE COURSE)****COURSE: U6CRMAT11: DISCRETE MATHEMATICS***(For Regular - 2014 Admission)*

Time: Three Hours

Max. Marks: 75

**PART A**Answer **all** questions. Each question carries 1 mark.

1. Define a regular graph.
2. Draw all simple graphs with two vertices.
3. Find the number of edges in  $K_{4,5}$ .
4. Draw a Hamiltonian graph which is not Euler.
5. Give an example of a graph with cut edge but no cut vertex.
6. Draw  $K_5$  and mark a maximum matching in the graph.
7. Encrypt the message RETURN HOME using Caesar cipher.
8. Write a super increasing Knapsack problem.
9. Define a chain.
10. State the Consistency law in lattice.

(1 x 10 = 10)

**PART B**Answer **any eight** questions. Each question carries 2 marks.

11. State and prove first theorem of graph theory.
12. Obtain Adjacency matrix of  $K_5$ .
13. Draw a graph which is not Euler but having an Euler trail and write the Euler trail.
14. Prove that it is impossible to have a group of nine people such that each one knows exactly five others in the group.
15. How many different perfect matching are there for the graph  $K_{n,n}$  for each  $n \geq 2$ ? Justify your answer.
16. Draw all trees with six vertices.
17. Encipher the message HAVE A NICE TRIP using Vigenere cipher with keyword MATH.
18. Obtain all solutions of the Knapsack problem  $2x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 + 11x_6 = 21$ .
19. Draw the lattice diagram of factors of 20 under divisibility.
20. Show that is any Lattice idempotent laws follow from absorption laws.

(2 x 8 = 16)

**PART C**Answer **any five** question. Each question carries 5 marks.

21. Show that an edge  $e$  of a graph  $G$  is a bridge if and only if  $e$  is not part of any cycle in  $G$ .

22. Show that a connected graph is Euler if and only if degree of every vertex is even.

23. If  $T$  is a tree with  $n$  vertices prove that it has precisely  $n-1$  edges.

24. Prove that  $k$ - regular bipartite graph with  $k > 0$  has perfect matching.

25. Let 'e' be an edge of a graph  $G$  prove that  $W(G) = w(G-e) w(G) + 1$ .

26. Decrypt the message RXQTGU which has enciphered by linear cipher  $c = 3p+7$ .

27. If  $(P, \leq)$  is poset with greatest element  $u$  such that every nonempty subset  $S$  of  $P$  has infimum then prove that  $P$  is a complete lattice.

(5 x 5 = 25)

### PART D

Answer **any two** questions. Each question carries 12 marks.

28. a) Prove that a non-empty connected graph  $G$  is bipartite if and only if it has no odd cycles.

b) If  $C(G)$  is complete then show that  $G$  is Hamiltonian.

29. Show that a matching  $M$  in a graph  $G$  is a maximum matching if and only if  $G$  contains no  $M$ -augmenting path.

30. A user of Knapsack cryptosystem has a private key consisting of the super increasing sequence

2, 3, 7, 13, 27. Modulus  $m = 60$  and multiplier  $a = 7$ .

(a) Find the user's listed public key.

(b) With the aid of this public key encrypt the message SEND MONEY.

31. (a) Prove that dual of a lattice is a lattice.

(b) Prove that a sub lattice  $S$  of a lattice  $L$  is a convex sub lattice if and only if for every  $a, b$  element of  $S$  with  $a \leq b$ ,  $[a, b] \subseteq S$ .

(12 x 2 = 24)

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