Reg. No.............................   Name.......................................   U604

B. Sc. DEGREE END SEMESTER EXAMINATION MARCH 2017

SEMESTR - **6: MATHEMATICS (CORE COURCE)**

COURSE: **U6CRMAT9 - U6CRCMT7, REAL ANALYSIS**

(For Regular - 2014 Admission)

Time: Three Hours   Max. Marks: 75

PART - A

Answer **all** questions. Each question has 1 mark

1. Show that $\sum\frac{n-1}{n+1}$ is not convergent
2. Show that the series $\sum(-1)^{n-1}$ oscillates
3. State Cauchy's Root test
4. Discuss the continuity of f(x)= $\frac{x-|x|}{x}$ if x≠0 and f (x)= 2 if x=0 at x=0
5. State intermediate value theorem
6. Define upper integral of a bounded real valued function on $[a, b]$
7. For an integer value i such that 0 ≤ i ≤5 let $x_i = \frac{i+i^2}{30}$ define P=$\{x_i : 0 \le i \le 5\}$ Find norm of P
8. Give an example of a function which is not Riemann integrable on a finite interval $[a, b]$
9. Define Uniform Convergence of a sequence of functions $\{f_n\}$
10. Show that Cos x + $\frac{Cos\ 2x}{2^2}$ + $\frac{Cos\ 3x}{3^2}$+............converges uniformly

(1 x 10 = 10)

PART – B

Answer **any eight** questions. Each question has 2 marks.

11. If $\sum u_n$ converges show that $\lim_{n\to\infty} u_n = 0$
12. Investigate the behavior of the series whose n^{th} term is Sin($\frac{1}{n}$)
13. Show that every absolutely convergent series is convergent
14. Prove that f(x)=$\frac{Sin2x}{x}$ if x≠ 0 and f(x)=0 if x=0 has a removable discontinuity at x=0
15. Prove that a function which is derivable at a point is necessarily continuous at that point
16. Show that the constant function is integrable
17. Give an example of a function f such that $\int_a^b |f|$ dx exists , but f is not integrable.
18. If f : $[a, b] \to$ R be continuous and bounded with f(x)≥ 0 for all x in $[a, b]$ then show that $\int_a^b f(x) \ge 0$
19. Show that $\{f_n\}$ where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in $[0, b]$; b> 0
20. Show that the sequence $\{f_n\}$ where $f_n(x)$= nx e^{-nx2}; x≥0 is not uniformly convergent on $[0, k]$; k> 0

(2 x 8 = 16)

PART - C

Answer **any five**. Each question has 5marks.

21. Examine the convergence of the series $\sum_1^{\infty} \sqrt{\frac{n}{n+1}}$ x^n

22. Define absolute convergence of a series $\sum u_n$. Also prove that the binomial series
$1+nx+\frac{n(n-1)}{1.2}x^2+$...........converges absolutely when $|x|<1$, n being a rational number

23. If a function is continuous on a closed interval show that it is bounded therein

24. Define Uniformly continuous functions. Prove that Uniformly continuous functions are continuous in that interval

25. Show that 3x+1 is Reimann integrable in $[1,2]$

26. If f_1 and f_2 are two bounded and integrable functions on $[a, b]$ prove that $f = f_1+f_2$ is also integrable on $[a, b]$

27. State and prove Cauchy's criterion for Uniform convergence

(5 x 5 = 25)

PART - D

Answer **any two**. Each question has 12marks**.**

28. (a) Prove that the positive term geometric series $1+r+r^2+r^3+$........ is convergent for $r <$ 1 and diverges for $r > 1$

(b) Test for convergence $\sum u_n$ where $u_n = \frac{n^{n^2}}{(n+1)^{n^2}}$

29. (a) Prove that a function f defined on an interval I is continuous at a point $c \in I$ iff for very sequence $\{c_n\}$ in I converging to c ,and $\lim_{n\to\infty} f(c_n)$=f(c)

30. (a) If f is bounded and integrable on $[a, b]$ prove that f^2 is also integrable

(b) Show that $f(x) = \frac{1}{2^n}$ where $\frac{1}{2^{n+1}} < x < \frac{1}{2^n}$ (n= 0, 1, 2,......) such that f(0) = 0 is integrable on $[0,1]$

31. (a) State and prove Weierstrass' M-test

(c) Show that the series $\sum_1^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ is uniformly convergent in every interval $[a, b]$

(12 x 2 = 24)

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